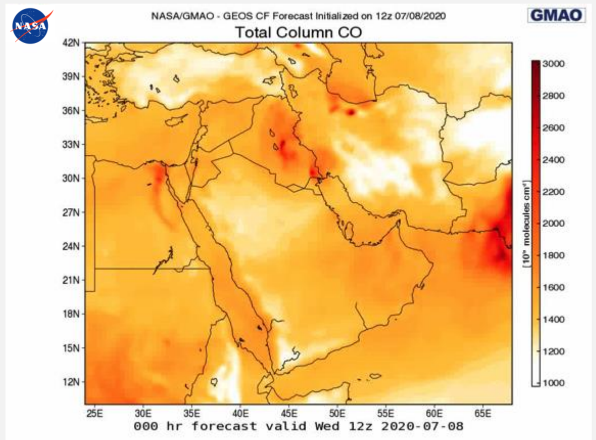
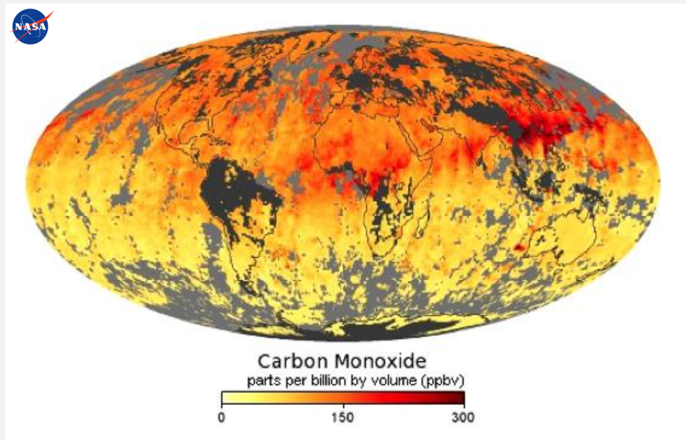
# Physics-Informed Neural Network Super Resolution for Advection-Diffusion Models

Chulin Wang, Eloisa Bentivegna, **Wang Zhou**, Levente J. Klein, Bruce Elmegreen
IBM Research

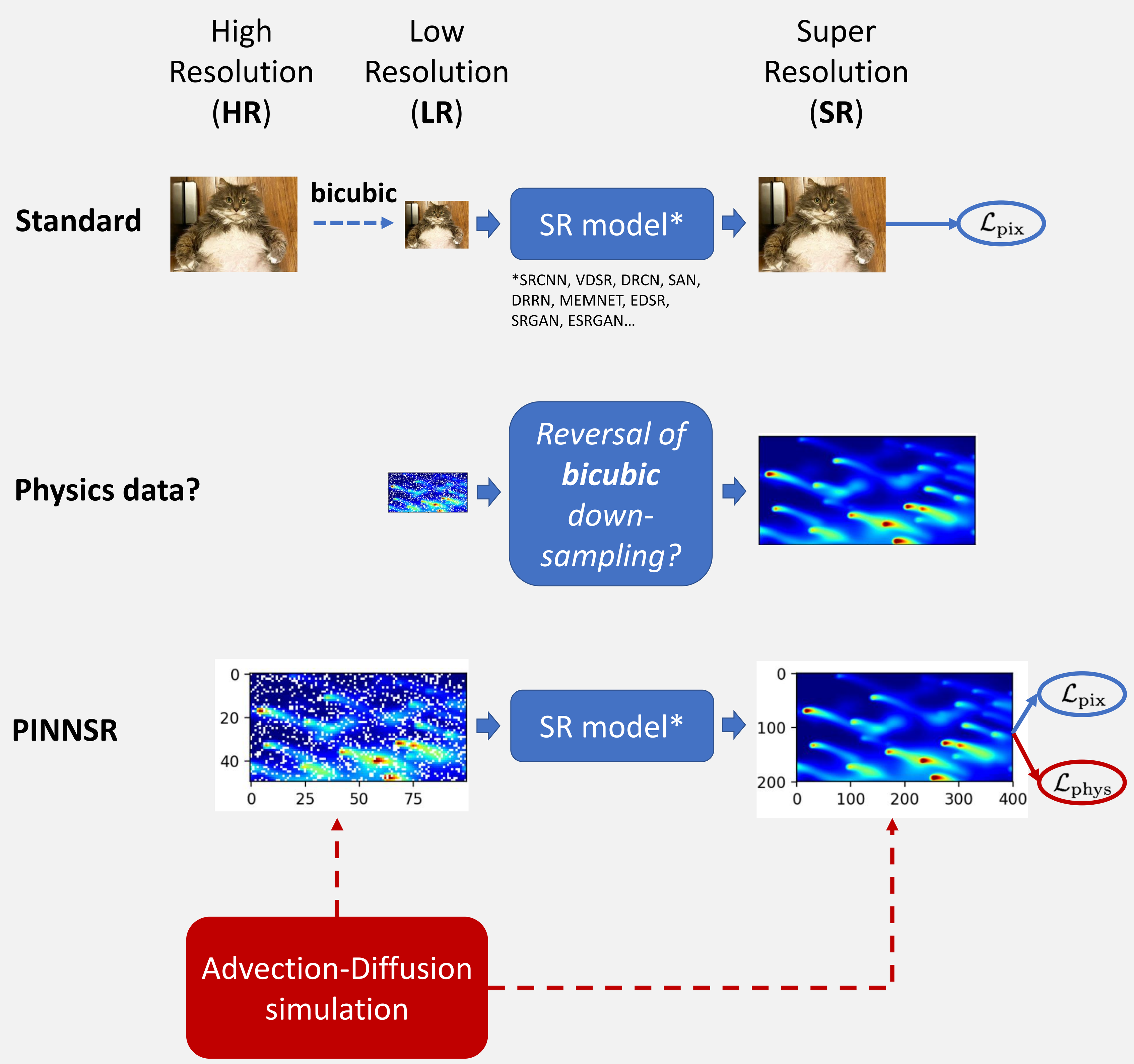
## Introduction

- Modeling of physical systems is often limited to coarse grid resolutions, typically in the scale of tens of kilometers per pixel.
- Real observations of the globe are sparse and often times incomplete (problem of missing pixels).
- Standard down-sampling approach to create low-resolution training data is not suitable for physics problems.
- We propose a Physics-Informed Neural Network for Super Resolution (**PINNSR**), which uses a *physics consistency loss* to guide the network.
- Randomly dropping pixels allows to simulate situations of sparse data.




## Standard SR v.s. PINNSR

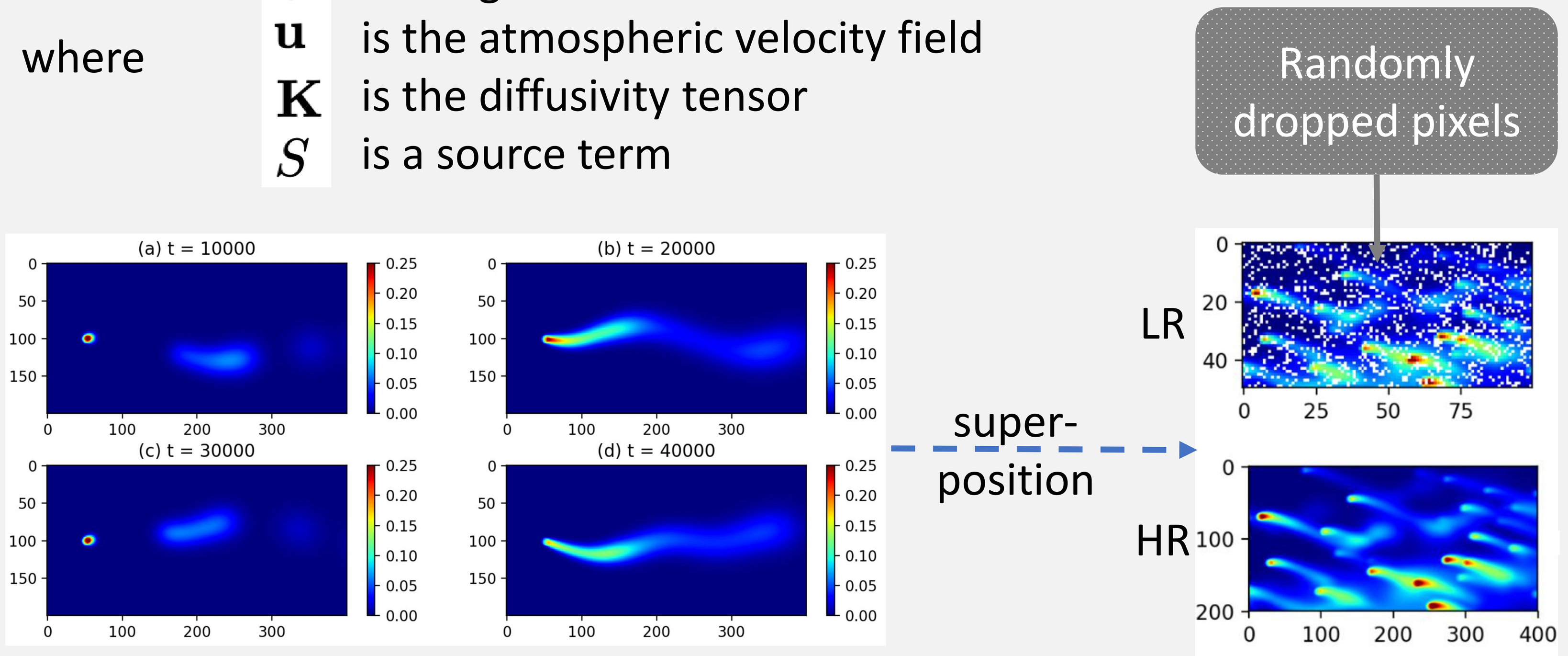



## Dataset

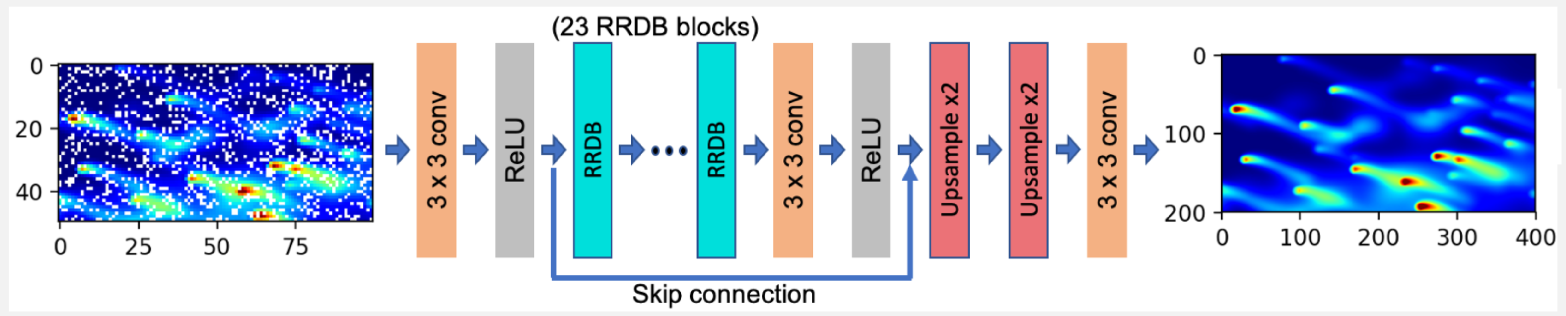
**Plume simulations based on an advection-diffusion equation**

$$\partial_t C + \nabla \cdot C\mathbf{u} = \nabla \cdot (\mathbf{K} \cdot \nabla C) + S$$

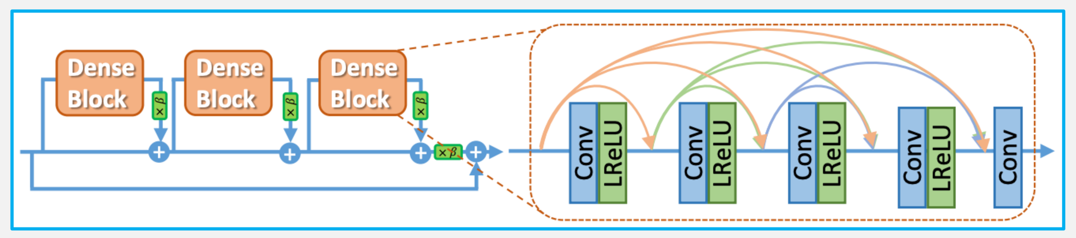
where
$C$ is the gas concentration
$\mathbf{u}$ is the atmospheric velocity field
$\mathbf{K}$ is the diffusivity tensor
$S$ is a source term




## Network




Residual in Residual Dense Block (RRDB)*

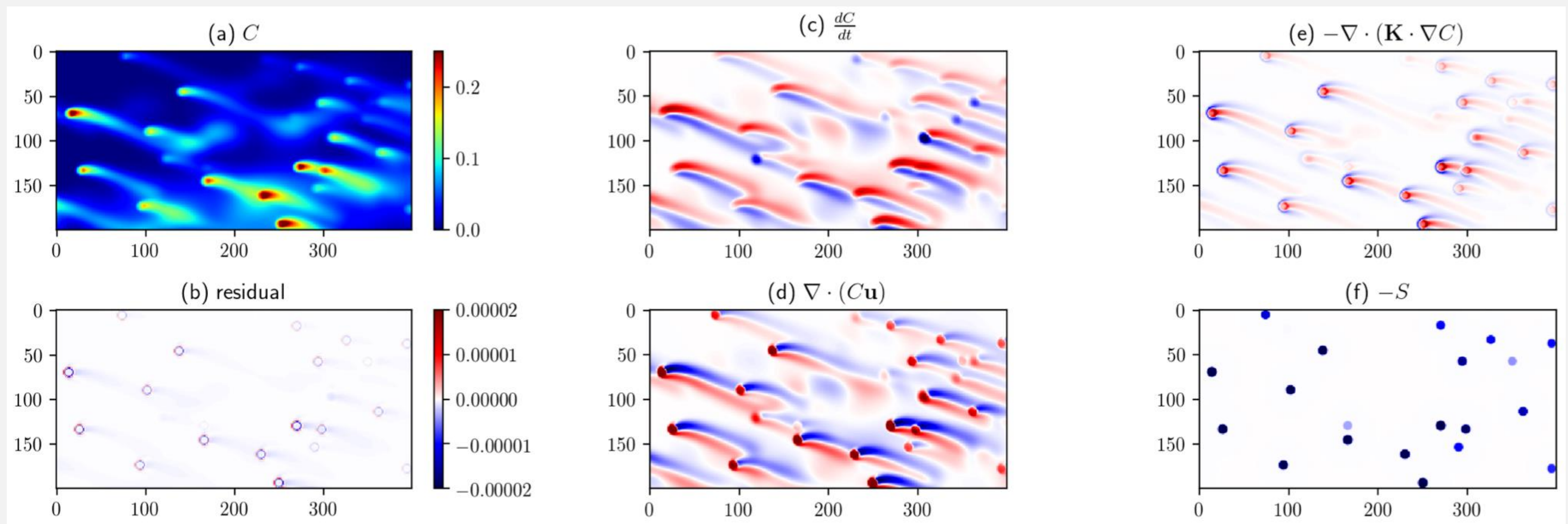



*Wang et al., "Esrgan: Enhanced super-resolution generative adversarial networks." In Proceedings of the European Conference on Computer Vision (ECCV), 2018.

## Physics Consistency Loss

$$R(C) = \partial_t C + \nabla \cdot C\mathbf{u} - \nabla \cdot (\mathbf{K} \cdot \nabla C) - S \quad \Rightarrow \quad \mathcal{L}_{\text{phys}} = ||R(C_{\text{SR}}) - R(C_{\text{HR}})||_1$$

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{pix}} + \eta \cdot \mathcal{L}_{\text{phys}}$$
$$= \frac{1}{N}\sum_{i=1}^{N} ||C_{\text{SR}} - C_{\text{HR}}||_1 + \eta \cdot \frac{1}{N}\sum_{i=1}^{N} ||R(C_{\text{SR}}) - R(C_{\text{HR}})||_1$$




## Results

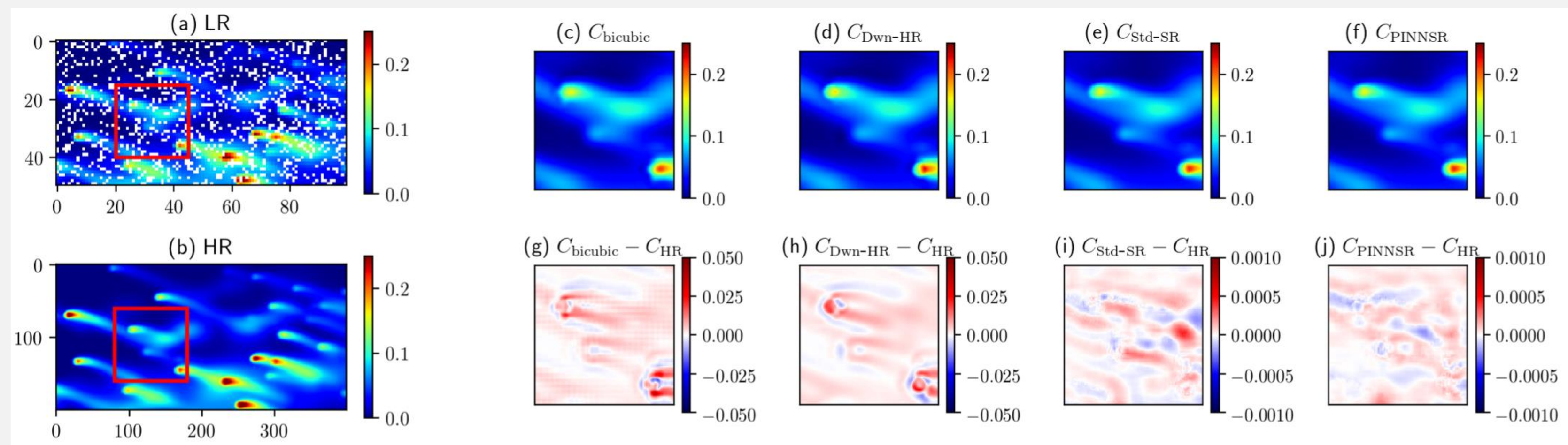
### Baselines

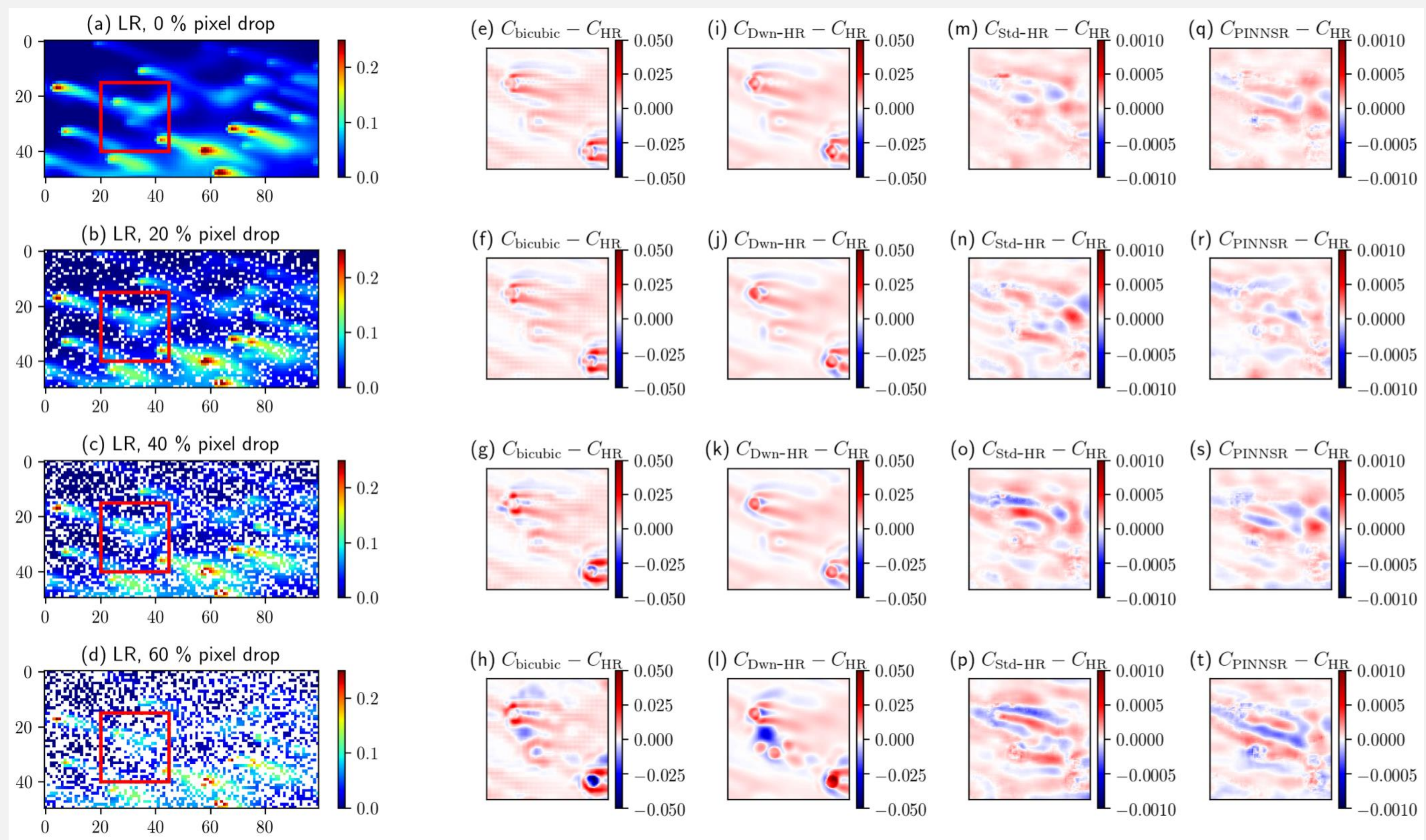
- **Bicubic** -- use bicubic interpolation to fill the missing pixels and 4× upsample
- **Dwn-HR** -- trained with down-sampled HR as input, test on simulated LR
- **Std-SR** -- trained on simulated LR but without the physics consistency loss
- **PINNSR** -- Our proposed approach

### Metrics

- Peak Signal-to-Noise Ratio (*PSNR*)
- Structural Similarity Index Measure (*SSIM*)
- Physics consistency loss

| | PSNR / 1 − SSIM / $\mathcal{L}_{\text{phys}}$ | | | |
|---|---|---|---|---|
| Pixel drop | Bicubic | Dwn-HR | Std-SR | PINNSR |
| 0% | 45.73 / 0.0065 / 5.9E-7 | 45.85 / 0.0058 / 1.1E-6 | 82.29 / 2.1E-6 / 2.8E-7 | **82.83 / 2.0E-6 / 0.9E-7** |
| 20% | 45.42 / 0.0070 / 6.4E-7 | 45.17 / 0.0064 / 1.5E-6 | 82.35 / **0.9E-6** / 1.8E-7 | **82.71** / 1.5E-6 / **1.2E-7** |
| 40% | 44.79 / 0.0080 / 7.2E-7 | 44.83 / 0.0065 / 1.1E-6 | 81.19 / 1.5E-6 / 3.0E-7 | **82.12 / 1.3E-6 / 1.5E-7** |
| 60% | 43.26 / 0.0128 / 8.8E-7 | 44.66 / 0.0061 / 0.8E-6 | 78.44 / 2.7E-6 / 4.1E-7 | **79.02 / 2.3E-7 / 2.6E-7** |




## Conclusion

- PINNSR with physics consistency loss shows superior performance on both reconstruction accuracy (11% improvement) and physics consistency.
- The method is robust even if pixels are missing as commonly observed in satellite images.

*Paper*