Solving high-dimensional parameter inference: marginal posterior densities & Moment Networks

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Marginal flows
- Full $p(x|\theta)$ density estimation for data $x$ and parameters $\theta$ intractable in high-dimension
- Make marginal densities the target of inference
- Estimate marginal posterior probability density for pairs of parameters $\alpha, \beta \in \theta$ by minimizing $-\sum \log q(\alpha, \beta, \mid x)$
- Result $q$ is an estimate (e.g. with normalizing flow) of marginal posterior $p(\alpha, \beta \mid x)$ if training parameters $\theta$, drawn from prior $p(\theta)$ and used to simulate $x$.

Markov chain Monte Carlo (MCMC) validation: see Figure A

LIGO-like gravitational wave time series
- Fig. B: two example simulated gravitational wave time series. The simplifed signals (dashed orange) are $\sim 0.12$s intervals from the 1 second before a binary black hole merger
- Fig. C left panel: Moment Network estimate of the marginal mean and variance for each time step parameter
- Fig. C right panel: validation case of similar complexity, but with known likelihood
- Trained Moment Network accurately matches long-run MCMC chain, which validates our approach

Cosmological applications
- Mapping the Universe: tractable simulation-based inference of high-dimensional cosmological fields & dark matter maps
- Robustness: Moment Networks use simpler architectures, reducing training failure risk and boosting inference speed
- Cross-validation: moments of estimated marginal posteriors should match those from Moment Networks
- Evading MCMC: Even when the likelihood is too complex to be sampled, marginal flows and Moment Networks still provide advantages, as many of the drawbacks of high-dimensional MCMC can be simply avoided

Highlights
- High-dimensional probability density estimation suffers from “curse of dimensionality”
- Propose direct estimation of lower-dimensional marginal posterior distributions
- Marginal flow: estimate joint-marginals of subsets of parameters
- Moment Networks: hierarchy of fast regression models compute increasing moments of lower-dimensional marginal posterior density
- Beyond likelihood-free inference: can also efficiently solve known MCMC problems
- Demo: high-dimensional inference: a) MCMC reference, b) Gravitational wave data model