Probabilistic Mapping of Dark Matter by Neural Score Matching

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Introduction

Dark Matter present in the Large-Scale Structure of the Universe is invisible, but its presence can be inferred through the **weak gravitational lensing** effect it has on the images of far away galaxies. By measuring this lensing effect on a large number of galaxies it is possible to reconstruct maps of the Dark Matter distribution on the sky. This, however, represents an extremely challenging **inverse problem** due to **missing data** and **noise dominated measurements**. In this work, we present a novel methodology for addressing such inverse problems by combining elements of Bayesian statistics, analytic physical theory, and a recent class of Deep Generative Models based on Neural Score Matching.

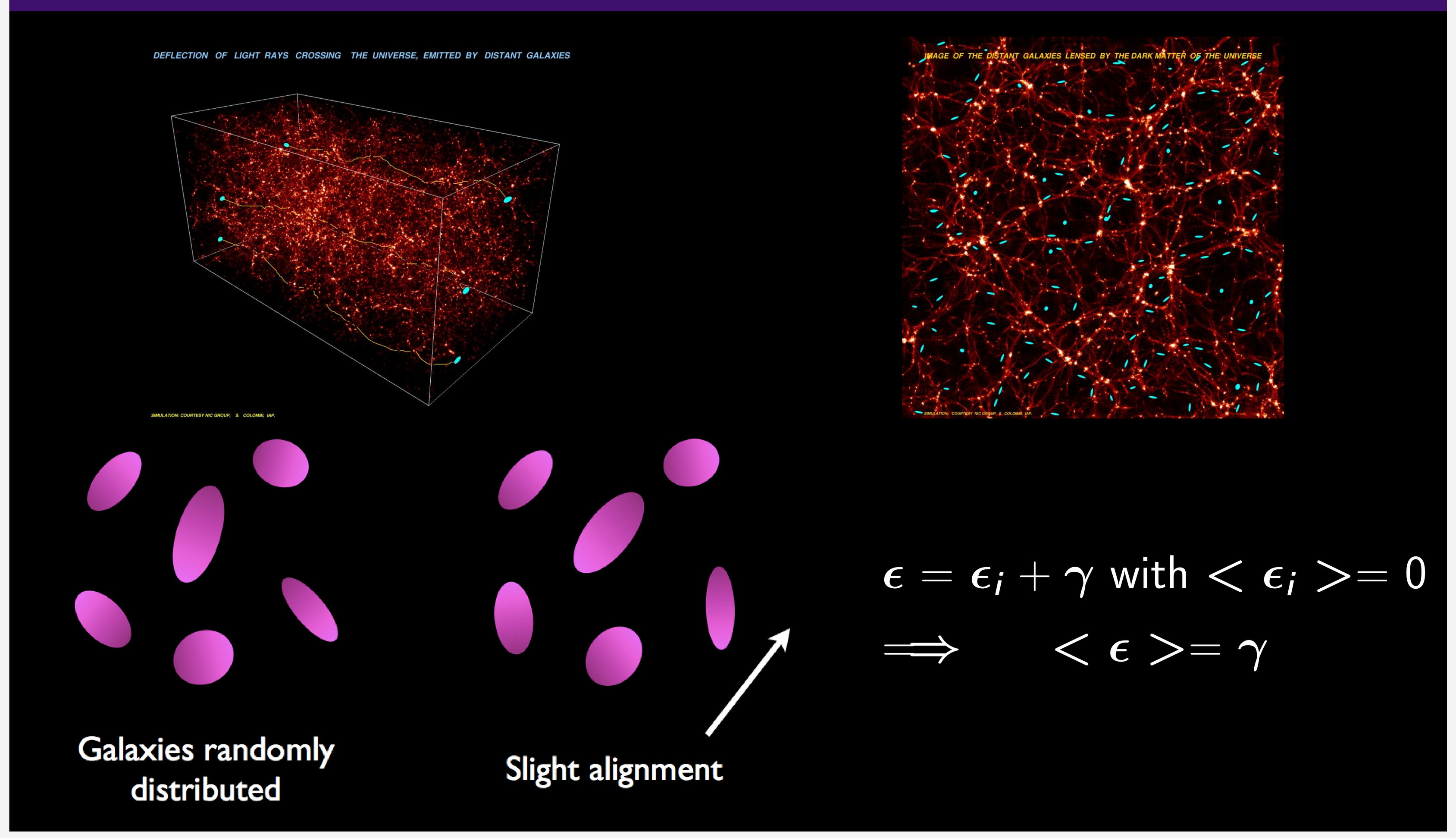
Our approach

Our approach allows to do the following:

1. Make full use of analytic cosmological theory to constrain the **2 point statistics** of the solution.
2. Learn from **cosmological simulations** any differences between this analytic prior and full simulations.
3. Obtain **samples** from the **full Bayesian posterior** of the problem for robust Uncertainty Quantification.

- We present an application of this methodology on the first **deep-learning-assisted Dark Matter map** reconstruction of the Hubble Space Telescope COSMOS field.

Weak Gravitational Lensing



Bayesian Inverse problem

Shear map γ and convergence map κ are related through the Kaiser-Squires (1993) transformation:

$$\gamma = \mathbf{MTPF}^*\kappa + \mathbf{n}, \qquad \mathbf{n} \sim \mathcal{N}(0, \sigma^2)$$

- It is an ill-posed inverse problem because of **missing data** and **noise corruption**.
- We aim to provide all the possible convergence map κ for a given observed ellipcitiy map ϵ, thus estimate the **posterior distribution**:

$$\underbrace{p(\kappa|\epsilon, \mathcal{M})}_{\text{posterior}} \propto \underbrace{p(\epsilon|\kappa, \mathcal{M})}_{\text{likelihood}} \underbrace{p(\kappa|\mathcal{M})}_{\text{prior}}$$

- The likelihood term $p(\epsilon|\kappa, \mathcal{M})$ encodes our physical understanding of the forward process that leads to the observation, given a set of cosmological parameters $\mathcal{M}$.

$$\log p(\epsilon|\kappa, \mathcal{M}) \propto -||\mathbf{M}(\gamma - \mathbf{TPF}^*\kappa)||^2_{\Sigma_n}$$

- The prior term $p(\kappa)$ encodes prior knowledge on the convergence map, given by analytic cosmological theory and learned on simulations.

Prior learning with Denoising Score Matching

- Prior on high dimensional images can be modeled **by learning the gradient of its log probability** $\nabla_x \log p(x)$, which is called the **score function** [1].
- Given a signal $x \sim p$, its noisy version $x' = x + n$, and $p_{\sigma^2} = p * \mathcal{N}(0, \sigma^2)$, an optimal denoiser $r^\star$ is related to the score function as [2], [3]:

$$r^\star(x', \sigma) = x' + \sigma^2 \nabla_x \log p_{\sigma^2}(x')$$

Hybrid prior

- We assume that the matter density field is gaussian at large scales. Then it is fully characterised by its 2 point statistics.

$$p_{th}(\kappa) = \frac{1}{\sqrt{\det 2\pi S}} \exp\left(-\frac{1}{2}\kappa^\dagger S^{-1}\kappa\right)$$

with S diagonal in Fourier space. Would yield only Gaussian constrained realisations or behave as a Wiener filter if the MAP is the target.

- Decomposition of the **score** of the full prior $p(\kappa)$:

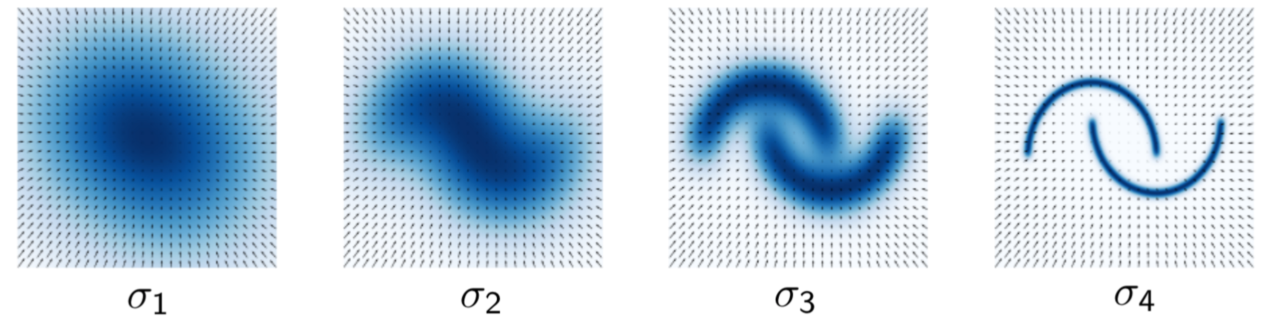
$$\underbrace{\nabla_\kappa \log p(\kappa)}_{\text{full prior}} = \underbrace{\nabla_\kappa \log p_{th}(\kappa)}_{\text{gaussian prior}} + \underbrace{r_\theta(\kappa, \nabla_\kappa \log p_{th}(\kappa))}_{\text{learned residuals}}$$

Sampling from score function

MCMC procedures (Langevin Dynamics, Hamiltonian Monte Carlo) only depends on the gradient of the log distribution.

$$x_{t+1} = x_t + \frac{1}{2}\epsilon \nabla_x \log p(x_t) + \sqrt{\epsilon} w, \quad w \sim \mathcal{N}(0, I)$$

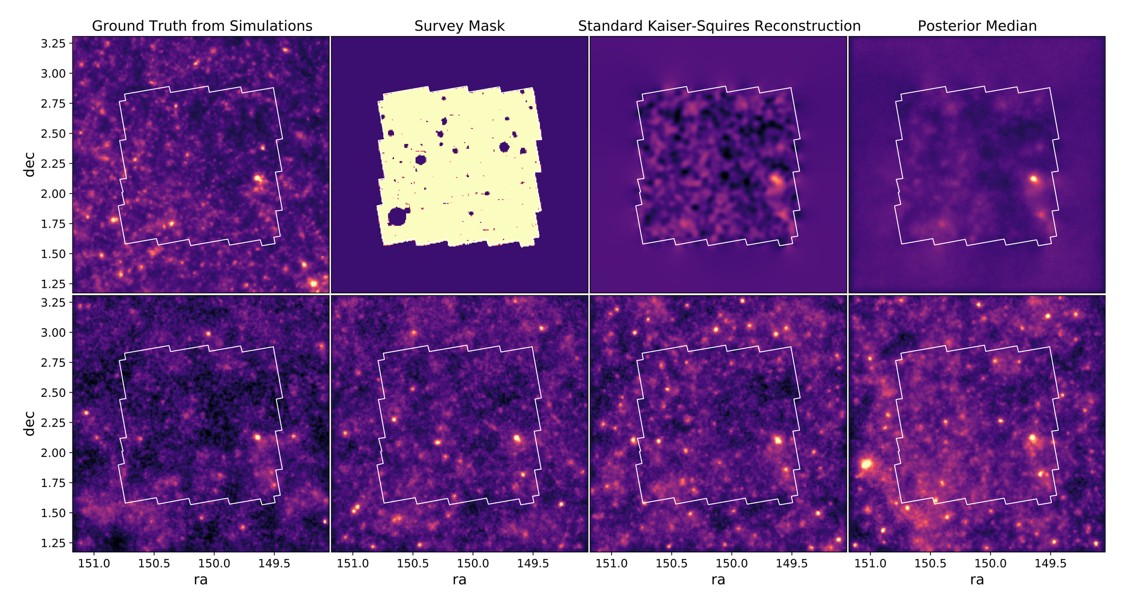
Annealing is used to avoid difficulties due to low density regions between modes. The MCMC updates are computed using a Gaussian-convolved version of the target density



σ^2 gradually annealed to low temperatures and the chain progressively moves towards a point in the target distribution. $\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4$, $p_{\sigma^2}(x) = \int p_{\text{target}}(t)\mathcal{N}(x|t, \sigma^2 I)$

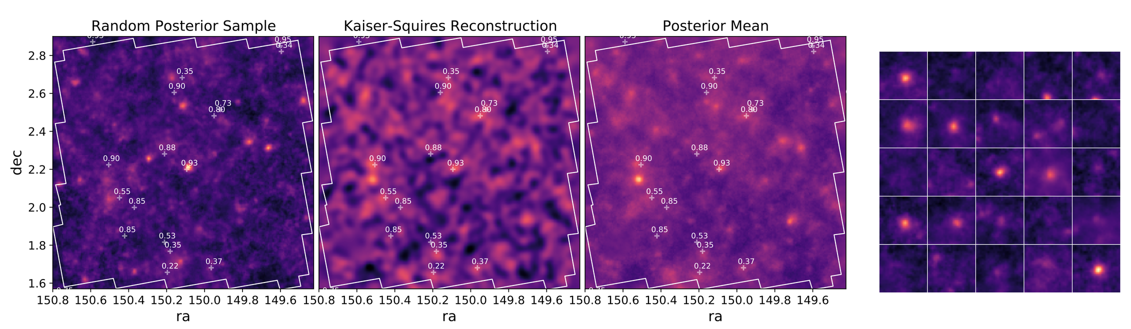
Results

- Training and testing on the `MassiveNus` suite of simulations [4].



Reconstruction of Dark Matter maps on simulated Hubble Space Telescope lensing measurements. Top row from left to right: true map from `MassiveNuS` simulations, COSMOS mask, a reconstructed map using a conventional KS method, and the median of our posterior samples. Bottom row shows samples from our estimated posterior.

- **High quality** mass map reconstruction from real survey [5].



Dark Matter map reconstruction of HST COSMOS survey. We compare a random sample of the posterior, the posterior mean and KS reconstruction. X-ray clusters and their redshifts are indicated in white.

References

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