Deep Potential: Recovering the gravitational potential from a snapshot of phase space


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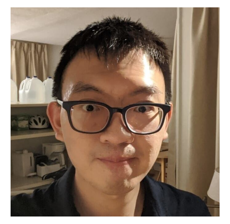
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One of the major goals of the field of Milky Way dynamics is to recover the gravitational potential field. Mapping the potential would allow us to determine the spatial distribution of matter – both baryonic and dark – throughout the Galaxy. We present a novel method for determining the gravitational field from a snapshot of the phase-space positions of stars, based only on minimal physical assumptions.

Definitions

- $\{\hat{x}, \hat{v}\}$ – phase-space observations (positions and velocities)
- $f(\vec{x}, \vec{v})$ – phase-space density, the *distribution function*
- $\Phi(\vec{x})$ – gravitational potential
- $\rho(\vec{x})$ – matter density

Physics

Stars are accelerated by the Galaxy's gravitational field: $\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = -\nabla\Phi(\vec{x})$

If we could measure stellar accelerations, then we could directly measure the gradients of the gravitational potential. Unfortunately, stellar accelerations due to the Galaxy's gravity are ~1 cm/s/yr, which is too small to measure. All we measure is a frozen snapshot of the positions and velocities of stars:

$$\{\hat{x}, \hat{v}\}$$

How to make progress? We assume that the Galaxy is stationary: the distribution function does not change in time. The collisionless Boltzmann equation links gradients of the potential and distribution function to the variation of the distribution function in time:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \sum_{\text{dimension } i} \left(v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} \right) = 0\,.$$

Our goal is to find the potential that renders the observed stellar population stationary.

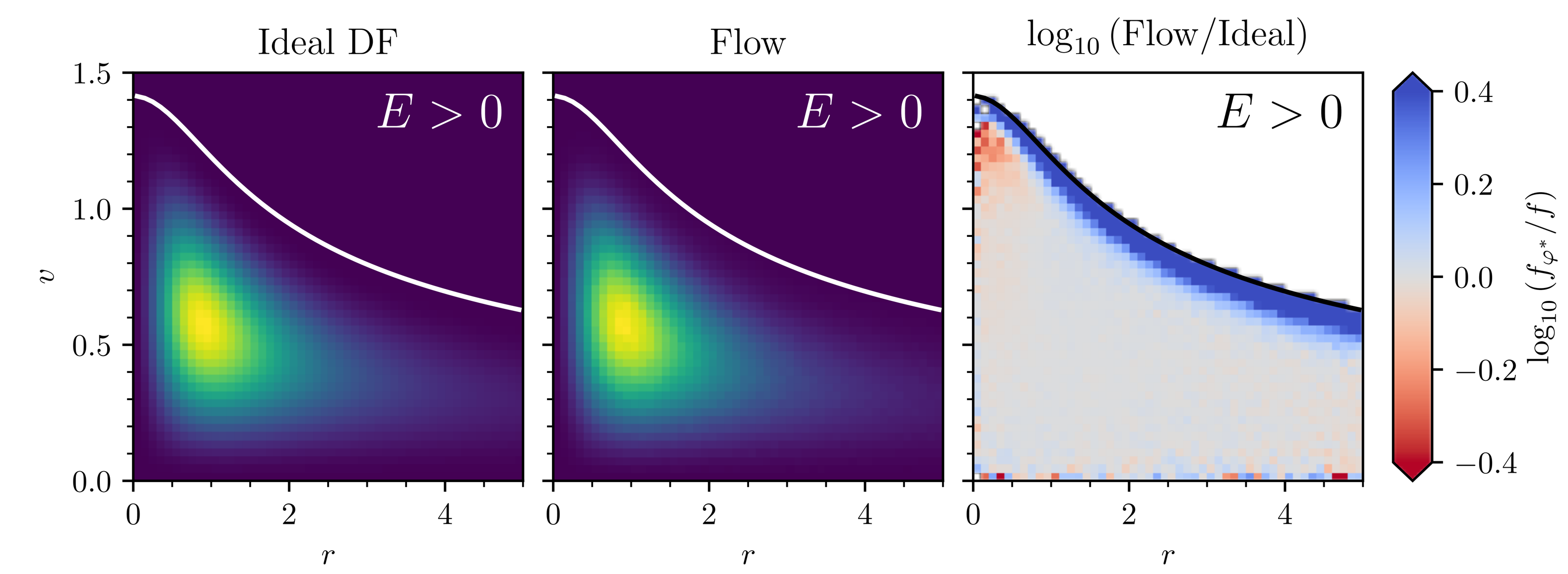
Demonstration on mock data

We test our method on mock data, using a toy physical model, the "Plummer sphere." The density and potential of the system are given by

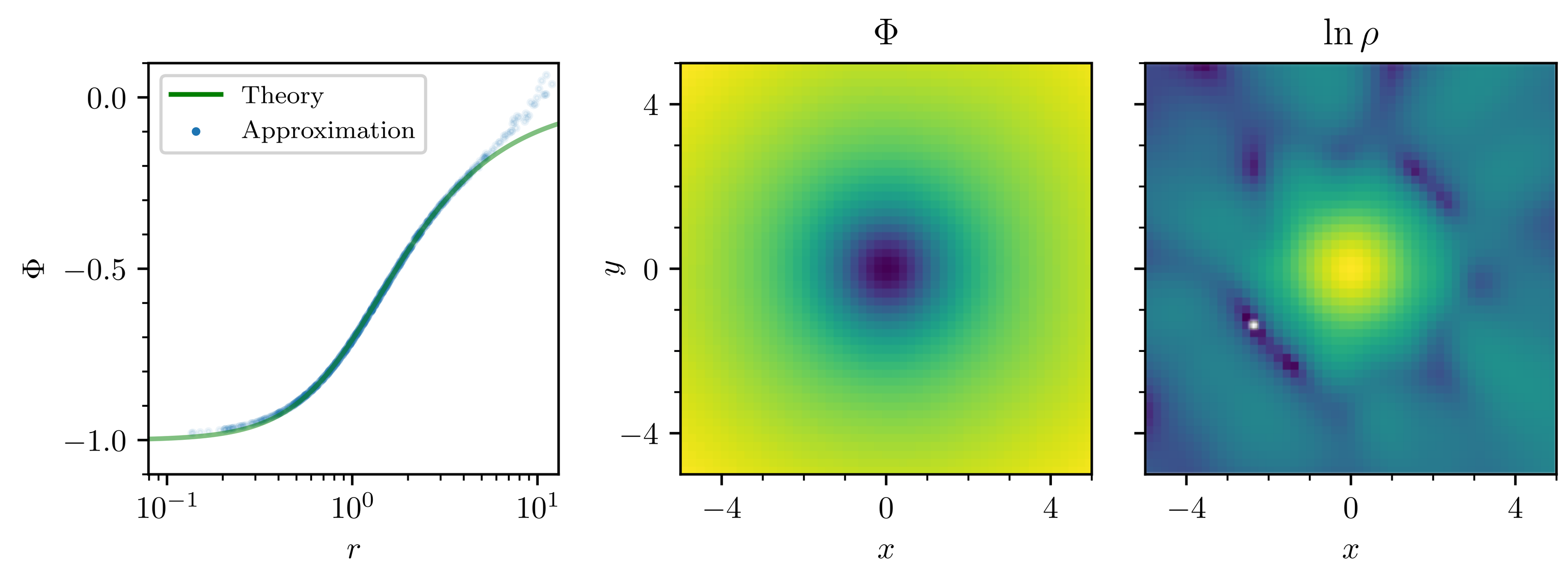
$$\rho(r) = \frac{3}{4\pi}\left(1+r^2\right)^{-\frac{5}{2}} \qquad \Phi(r) = -\left(1+r^2\right)^{-\frac{1}{2}}$$

We draw 2^{17} (131,072) positions and velocities from this model, and feed them into our machinery.

Below: The ideal Plummer sphere distribution function (*left panel*), our trained ensemble of normalizing flows (*middle panel*), and a comparison of the two (*right panel*). We depict phase space in terms of radius and velocity, integrating over the four angular dimensions. Our ensemble of normalizing flows performs well in regions of non-negligible density.



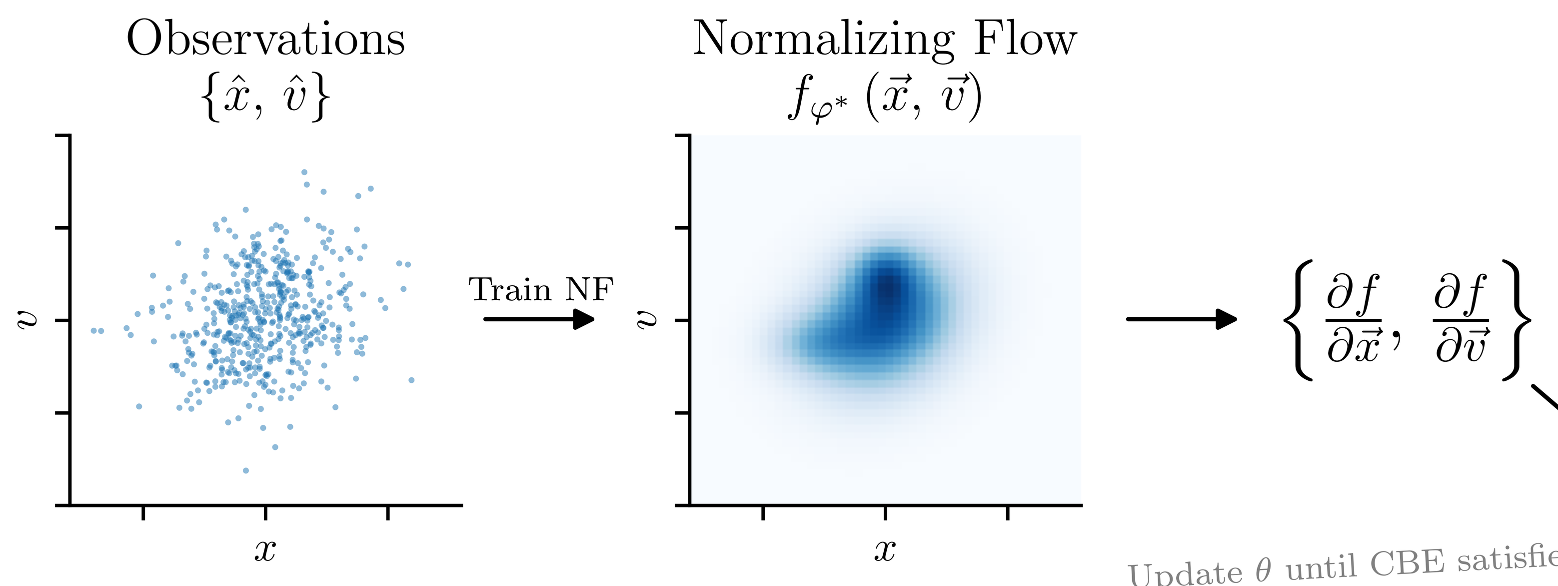
Below: The left panel compares the theoretical Plummer sphere potential with our result at random points drawn from phase space. The middle and right panels show our recovered potential and matter density in a 2D slice of space with z = 0.



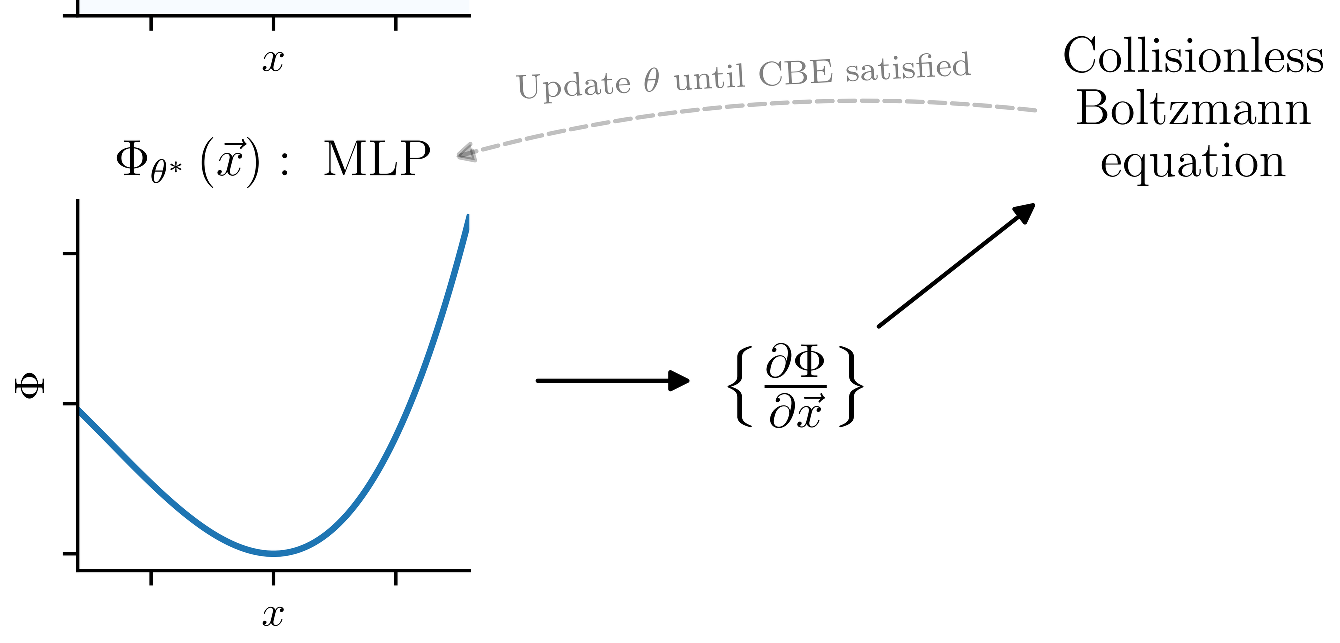
We recover the potential with high accuracy over a wide range of radii, and accurately trace the density in the core of the system, where most of the mass resides.

Machine learning

1. We fit a normalizing flow to the observed stellar positions and velocities, obtaining a smooth, differentiable approximation of the distribution function.



3. We can evaluate the stationarity of the recovered distribution function by plugging its gradients and those of the potential into the collisionless Boltzmann equation



2. We represent the gravitational potential by a multi-layer perceptron. We update its parameters until its gradients render the recovered distribution function stationary.

Future directions

The Gaia space telescope is precisely measuring positions and velocities of over a billion stars in the Milky Way. By applying Deep Potential to this data, we aim to uncover the distribution of mass in our Galaxy.

Code

PyTorch implementation:
https://github.com/tingyuansen/deep-potential

Tensorflow 2.x implementation:
https://github.com/gregreen/deep-potential

Acknowledgments

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