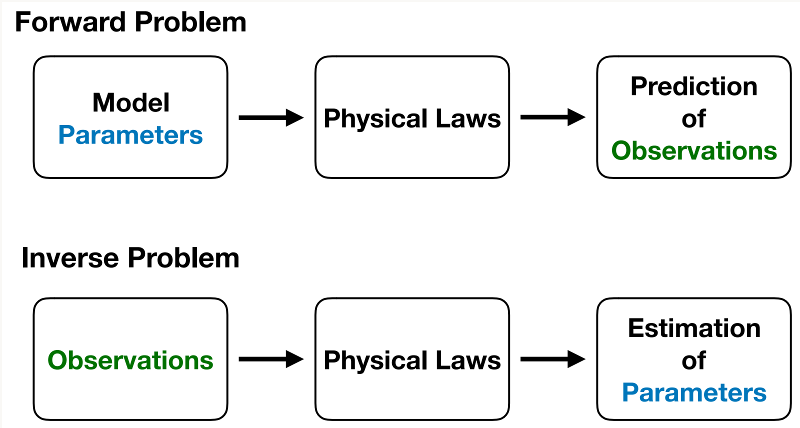
ADCME: Learning Spatially-varying Physical Fields using Deep Neural Networks

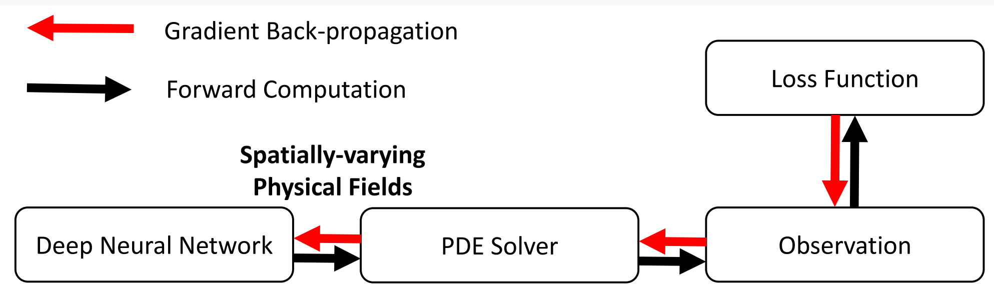
Background



Forward Problem and Inverse Problem in Computational Engineering.

- Modeling complex physical processes involves coupling many physical laws/models, in which many of them are unknown.
- Availability of **experimental data and observations** enables us to build data-driven physical models.
- **Deep neural networks** emerge as an empirically successful function approximator for complex and high dimensional functions.
- We propose an approach to couple **numerical solvers and deep neural networks** for data-driven inverse modeling.

Workflow



Methodology

The PDE-constrained optimization formulation for inverse problem:

$$\min_{\theta} L_h(u_h) \quad \text{s.t. } F_h(\theta, u_h) = 0$$

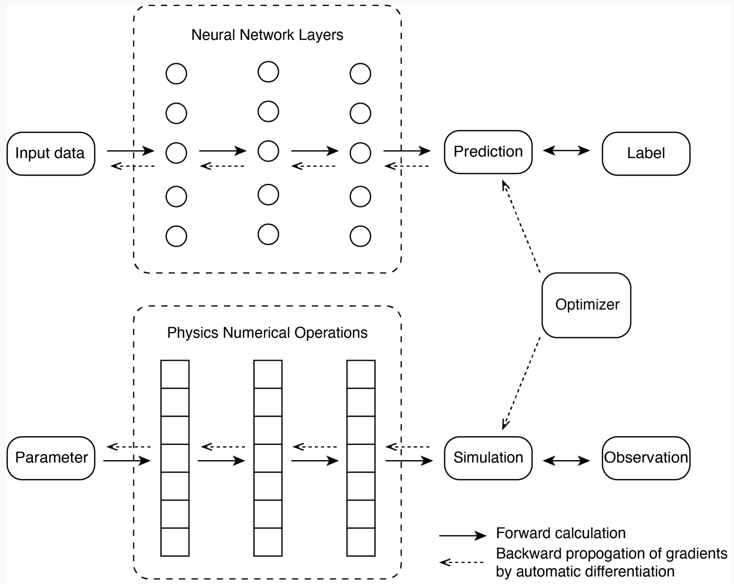
1. Approximate the unknown function using a deep neural network;

$$\min_{\theta} L_h(u_h) \quad \text{s.t. } F_h(NN_\theta, u_h) = 0$$

2. Reduce the constrained optimization problem to an unconstrained one using a **numerical solver** (e.g., FEM);

$$\min_{\theta} \tilde{L}_h(\theta) := L_h(u_h(\theta))$$

3. Express both numerical solvers and deep neural networks as computational graphs;



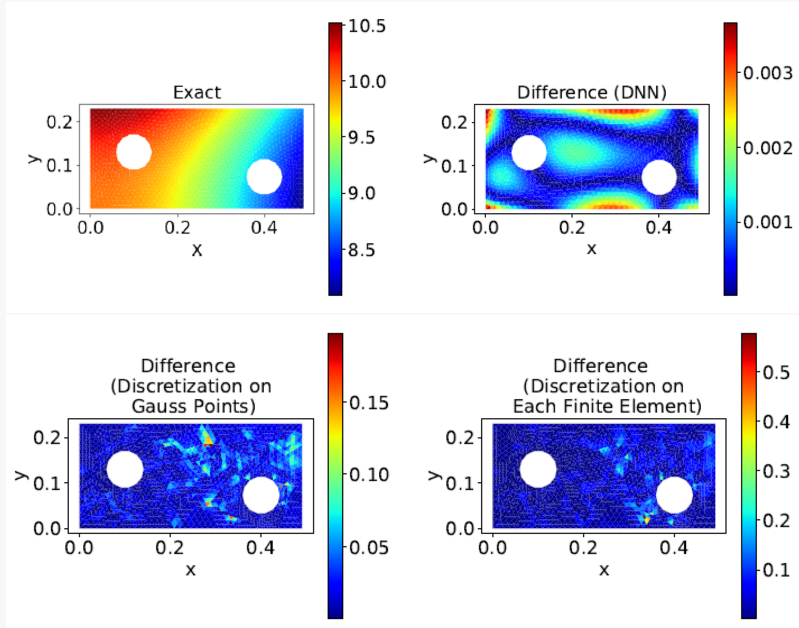
4. Calculate the gradients using reverse-mode automatic differentiation.

Software: https://github.com/kailaix/ADCME.jl

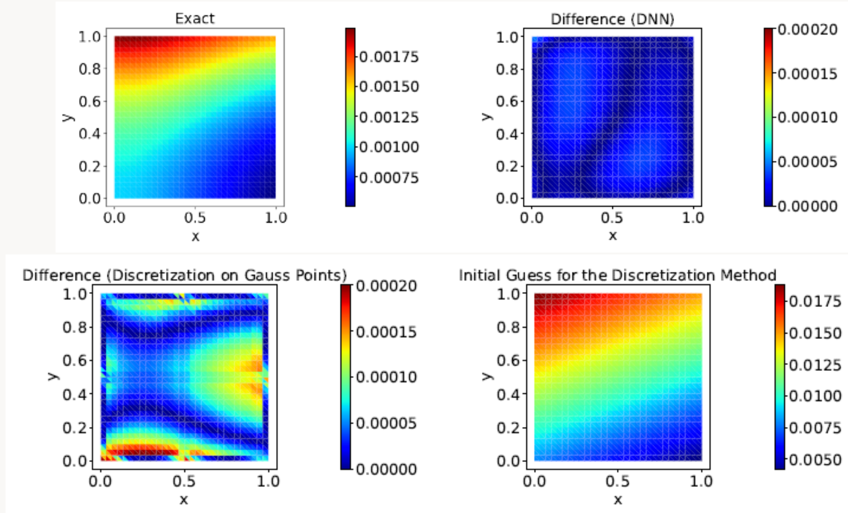
Result

- Hyperelasticity
  - Estimating Young's modulus from the displacements of the material.

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$



- Burgers' Equation
  - Estimate the viscosity parameter from velocities.



For more details: https://arxiv.org/abs/2011.11955

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