

Introduction

Underground analysis of fractured media requires flow simulations (e.g. applications geothermal applications, oil & gas production).

Discrete Fracture Networks (DFN) are discrete models composed by a network of 2D polygonal fractures in a 3D domain, that can accurately simulate the flow of a fracture medium.

The reformulation by [Berrone, Pieraccini, Scialo' 2013, 2014, 2016] guarantees advantages but numerical solutions of DFN are still prohibitive for the large number of simulations required by Uncertainty Quantification (UQ) analyses.

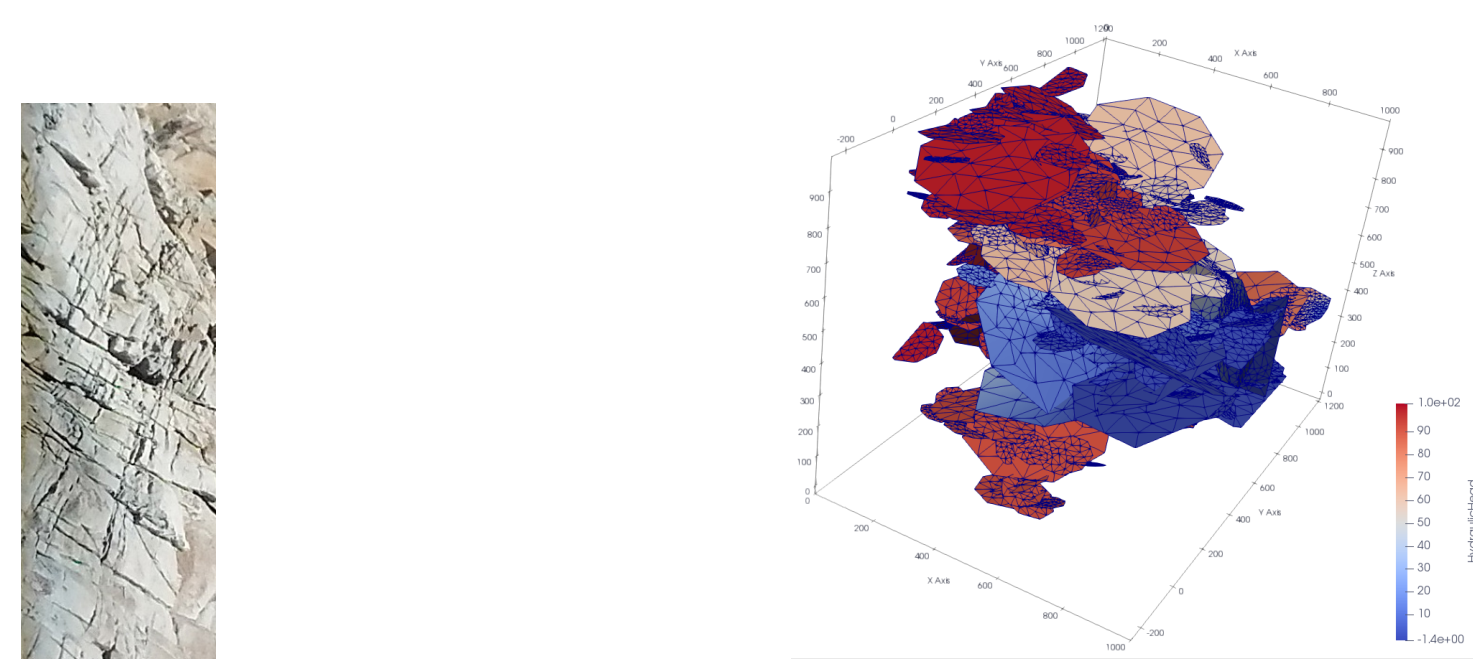
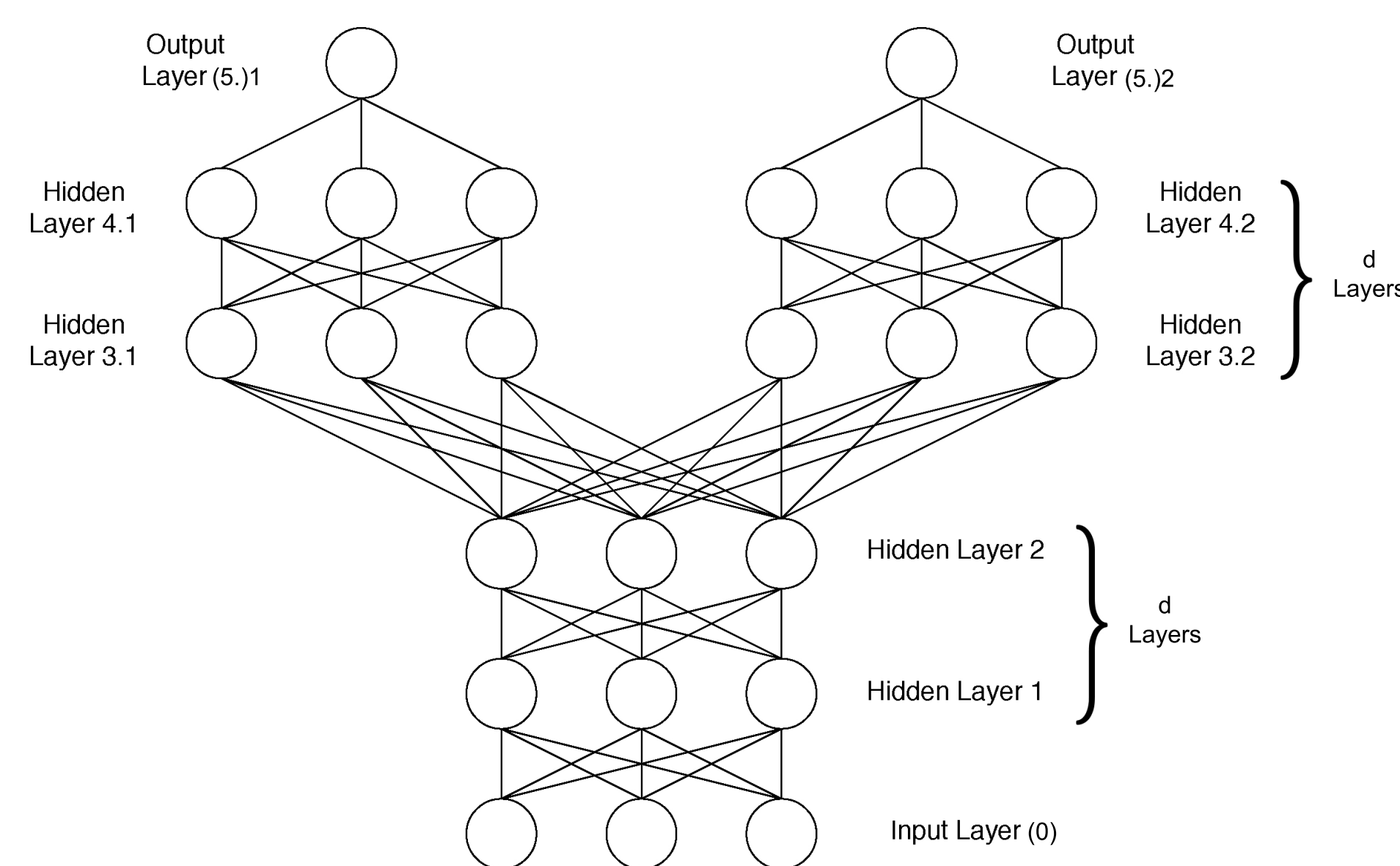


Figure 1: surface of a natural fractured medium (left) and a DFN (right)

NN for Flux Regression in DFN

Use a **Neural Network (NN)** for regression of **exiting fluxes** $\phi \in \mathbb{R}^m$ from DFN158



- Neural Network** Fully connected multi-headed, tree-shaped architecture, trunk and branches depth 3, 158 units \times layer, soft-plus activation, Adam optimizer, early stopping with patience 150.

	\mathcal{F}_8	\mathcal{F}_{12}	\mathcal{F}_{14}	\mathcal{F}_{78}	\mathcal{F}_{90}	\mathcal{F}_{98}	\mathcal{F}_{107}
D_{KL}/\mathcal{E}	0.0009	0.0003	0.0010	0.0002	0.0033	0.0379	0.0010

Table 1: Dissimilarity between ϕ , $\hat{\phi}$, actual and predicted outflux distributions; D_{KL} : KL divergence between ϕ , $\hat{\phi}$; \mathcal{E} : entropy of ϕ

Major Issues

Fracture media cannot be fully described, then:

- Generation:** lacks of full deterministic data
 \Rightarrow **DFNs stochastically generated.**

Quantify the uncertainty of stochastic generation
 \Rightarrow **Uncertainty Quantification (UQ)**

- Simulation:** **complex** computational domain & **expensive** computations

Reduce the DFN complexity
 \Rightarrow **Backbone Identification**

Main Target: Backbone Identification

Backbone B: sub-network of fractures with **transport characteristics** approximating the original DFN

- DFN158:** Fix the DFN geometry with $n = 158$ fractures randomly generated from geological distributions (7 outflow fractures). Assume varying fracture transmissivities $\log_{10} \kappa_i \sim \mathcal{N}(-5, 1/3)$.
- Flow Simulation:** fixed boundary Dirichlet conditions of fixed pressure ΔH between influx and outflux fractures.
- Backbone validation:** run flow simulations of fractures sub-network and compare ϕ , ϕ_B .

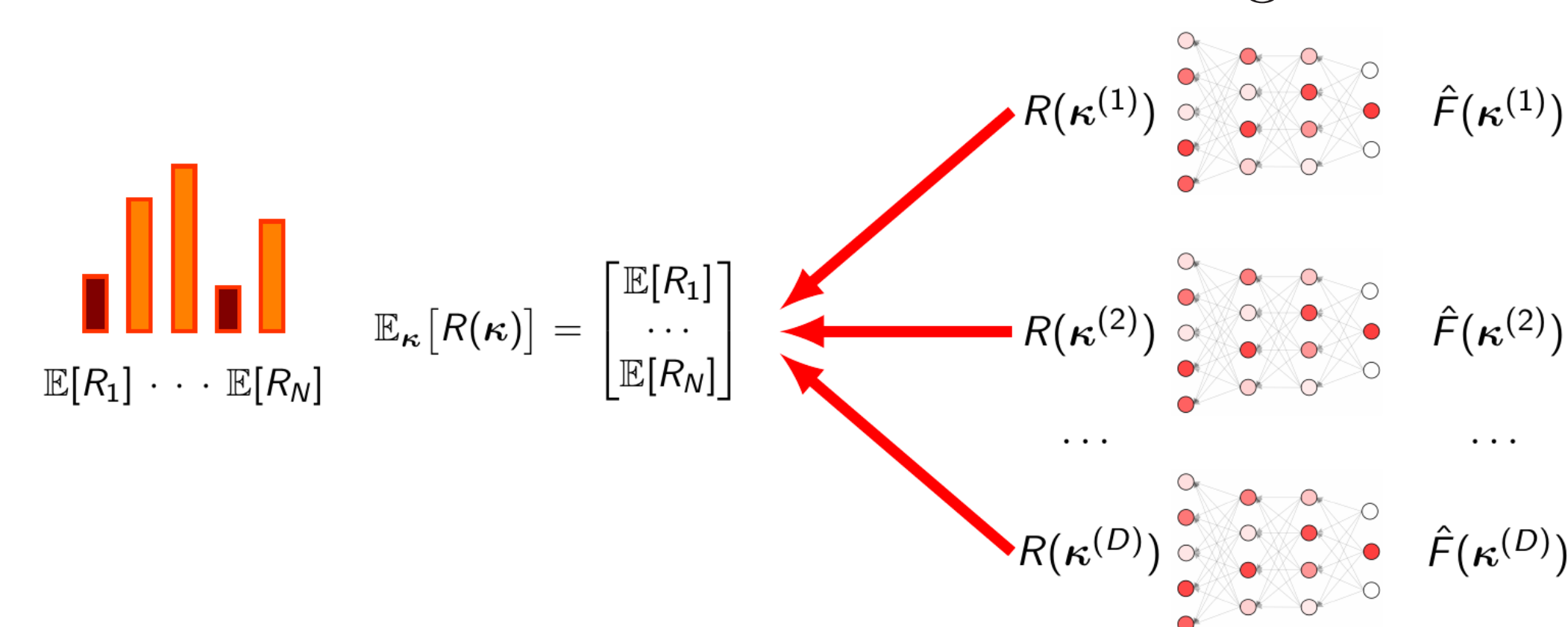
ϕ , ϕ_B exiting flux distributions of full DFN and Backbone:
 $\Rightarrow \phi \approx \phi_B$

LRP for Backbone Identification

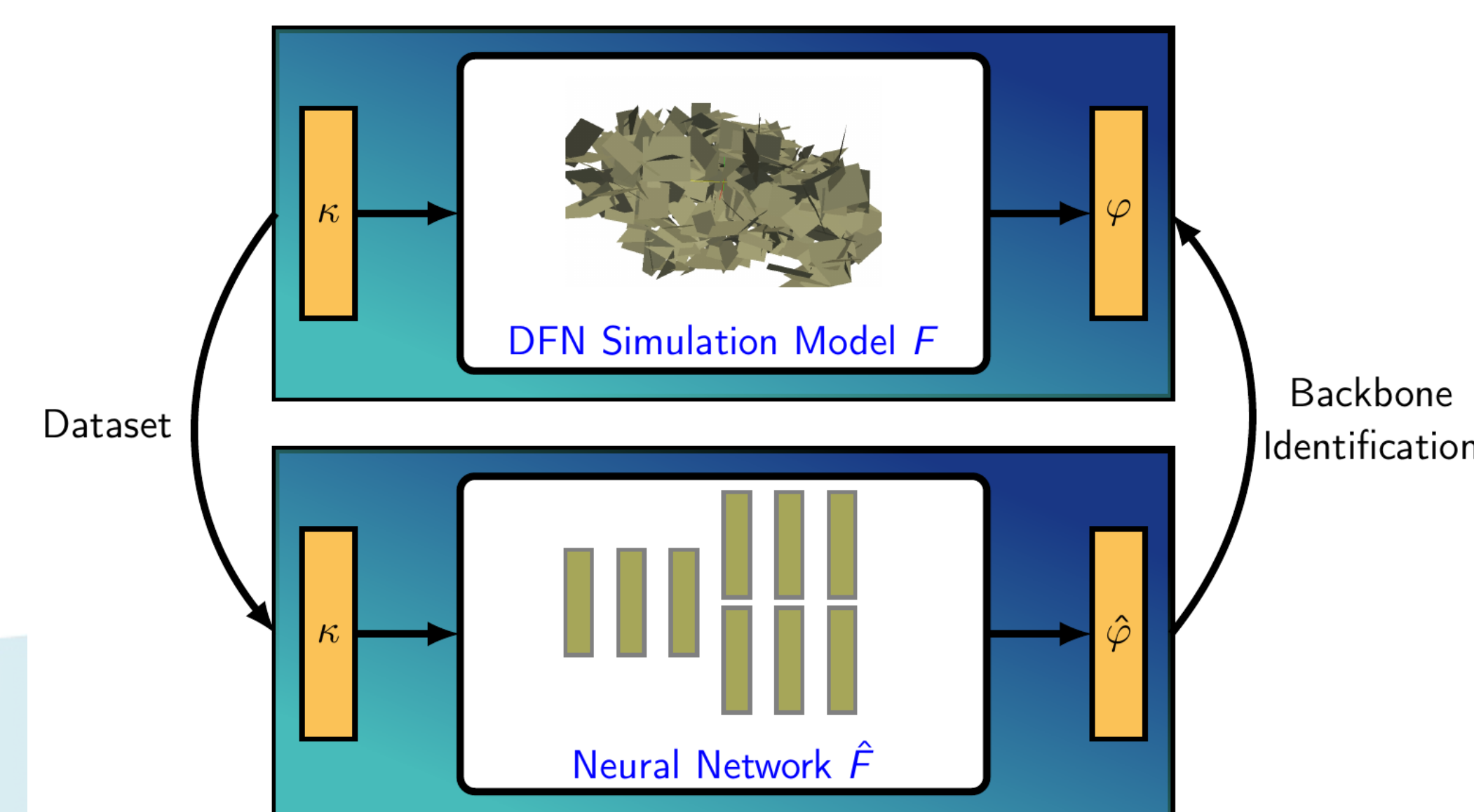
- Local algorithm of eXplainable AI**
Layer-wise Relevance Propagation (LRP) [Bach, 2015]:

$$R_i^{(\ell)} = \sum_{j \in (\ell+1)} R_{i \leftarrow j}^{(\ell, \ell+1)}, \quad \text{neuron } i \in \ell \text{ layer.}$$

- Here extend to **global explanation:**
Expected Relevance as a **feature selection** algorithm:



- Overall pipeline for **Backbone Identification:**



Results

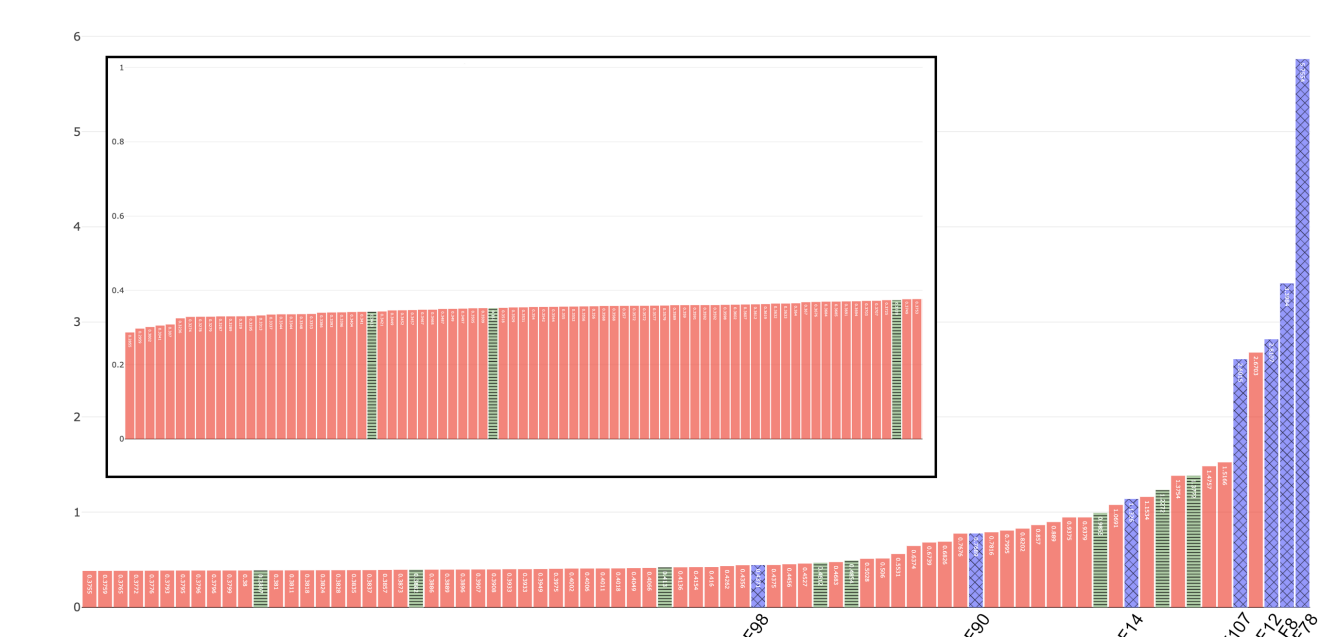


Figure 2: Fractures ordered by ascending value of r (top-left corner: lowest 60%); (blue) labelled outflow fractures; (green) inflow fractures. Outflow fractures are in the top-25% of expected relevance
 \Rightarrow **NN approximates fluxes coherently with the DFN topology.**



Figure 3: Graphs of DFN158 (top-left) and Backbones with top expected relevance: 75% (top-right), 50% (bottom-left), 25% (bottom-right). **NN seems understanding that some bottleneck nodes are fundamental:** a source-sink path is kept for the backbone top 25% expected relevance.