Targeted Likelihood-Free Inference of Dark Matter Substructure in Strongly-Lensed Galaxies

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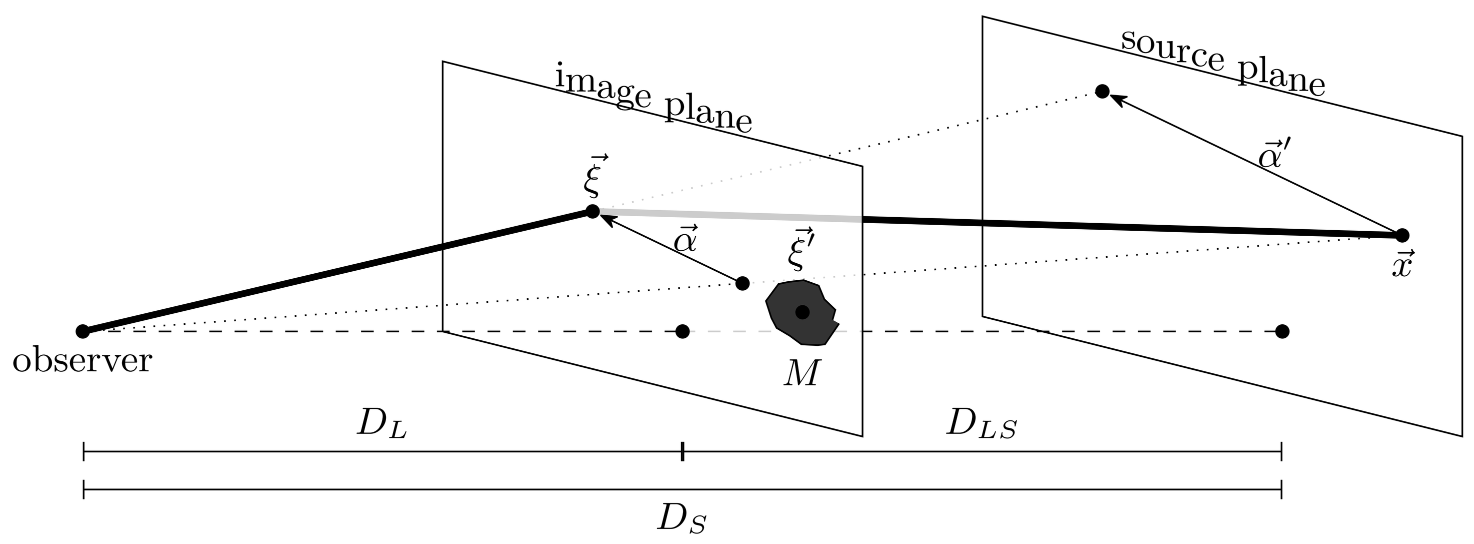
Lensing and dark matter substructure

Strong gravitational lensing uniquely probes **dark matter subhalos**



Goal: compute posteriors for subhalo position and mass

Difficulties: complex correlations between image pixels; marginalization over many model parameters & subhalos

Methodology:

1. Fit ***approximate posterior*** for source and lens model parameters to an observation. Posterior acts as a ***targeted simulator*** to producing data similar to the observation.
2. Train ***likelihood-free inference*** network to compute substructure posteriors, marginalizing out model parameters

A new model for gravitational lenses

Standard GP

$\mathbf{f} \sim N(0, \mathbf{K})$

$\mathbf{x} \sim N(\mathbf{f}, \sigma_n^2)$

$\longrightarrow$ $\mathbf{K} = \alpha^2 \mathbf{T}\mathbf{T}^\top$

Our model

$\mathbf{y} \sim N(0, \alpha^2)$

$\mathbf{f} = T(\mathbf{p}, \sigma)\, \mathbf{y}$

$\mathbf{x} \sim N(\mathbf{f}, \sigma_n^2)$

$\mathbf{f}$: true fluxes in each pixel

$\mathbf{x}$: observation

σ_n : observation noise

$\mathbf{y}$: source parameters (1/pixel)

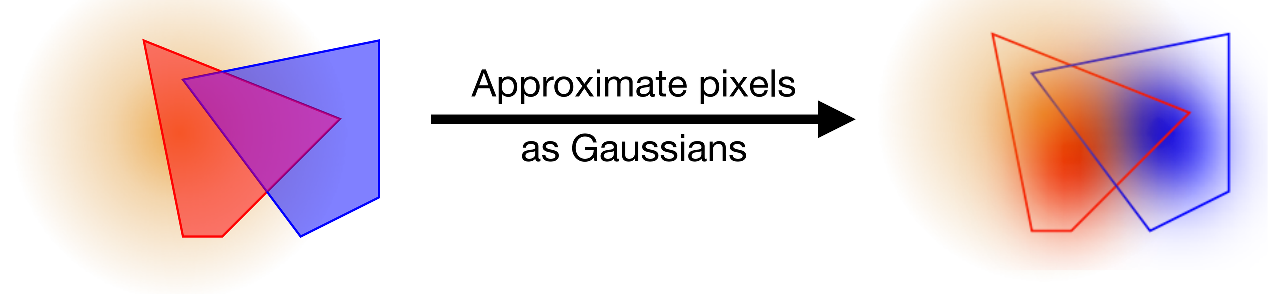
α : variance hyperparameter

σ : kernel size hyperparameter

$\mathbf{p}$: pixel coordinates

Covariance $\mathbf{K}$ induced by **intrinsic source variations** & **pixel overlaps in source plane:**

$$\mathbf{K}_{ij} = \iint d\mathbf{p}_1\, d\mathbf{p}_2\, g_i(\mathbf{p}_1)\, k(\mathbf{p}_1, \mathbf{p}_2)\, g_j(\mathbf{p}_2)$$



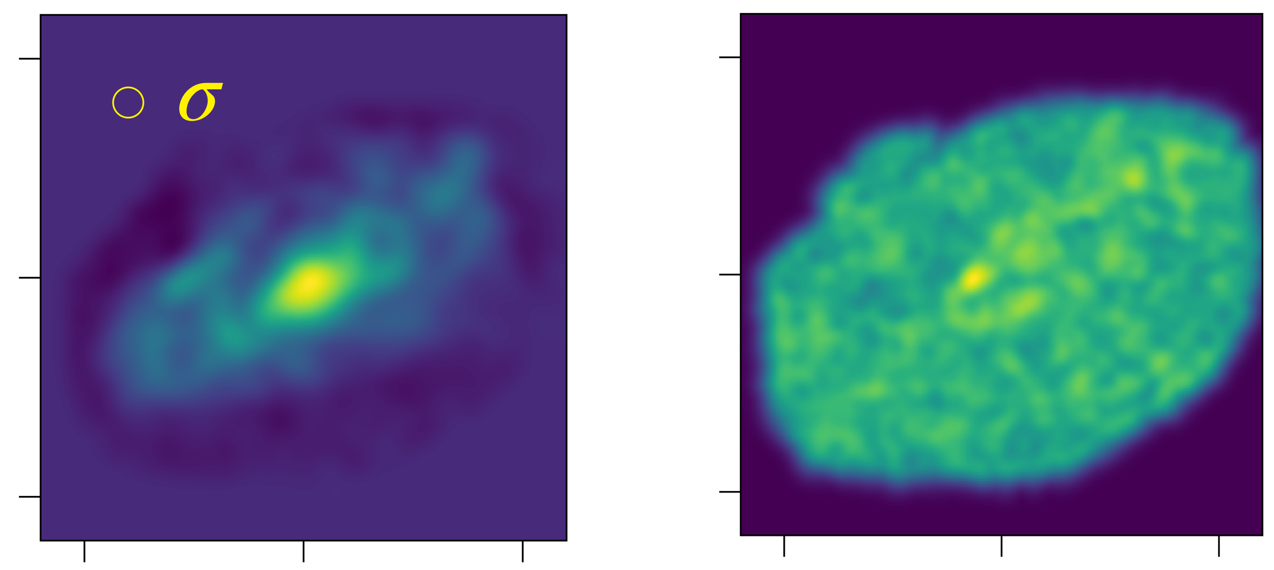
Can approximate $\mathbf{T}$ using fact that matrix multiplication ~ spatial convolution

- Instead of infeasible GP matrix inversion, fit **variational posterior** for lens & source parameters to observation, *ignoring substructure*
- **ELBO maximization** via gradient descent with reparametrization trick (using `pyro`)

➡ Requires lens model to be automatically-differentiable (implemented in `pytorch`)

- Simultaneously optimize α hyperparameter with σ fixed by hand
- We use a diagonal-normal posterior for the ~10^5 source parameters & multivariate normal for lens parameters

Source mean | **Source uncertainty**



Substructure inference

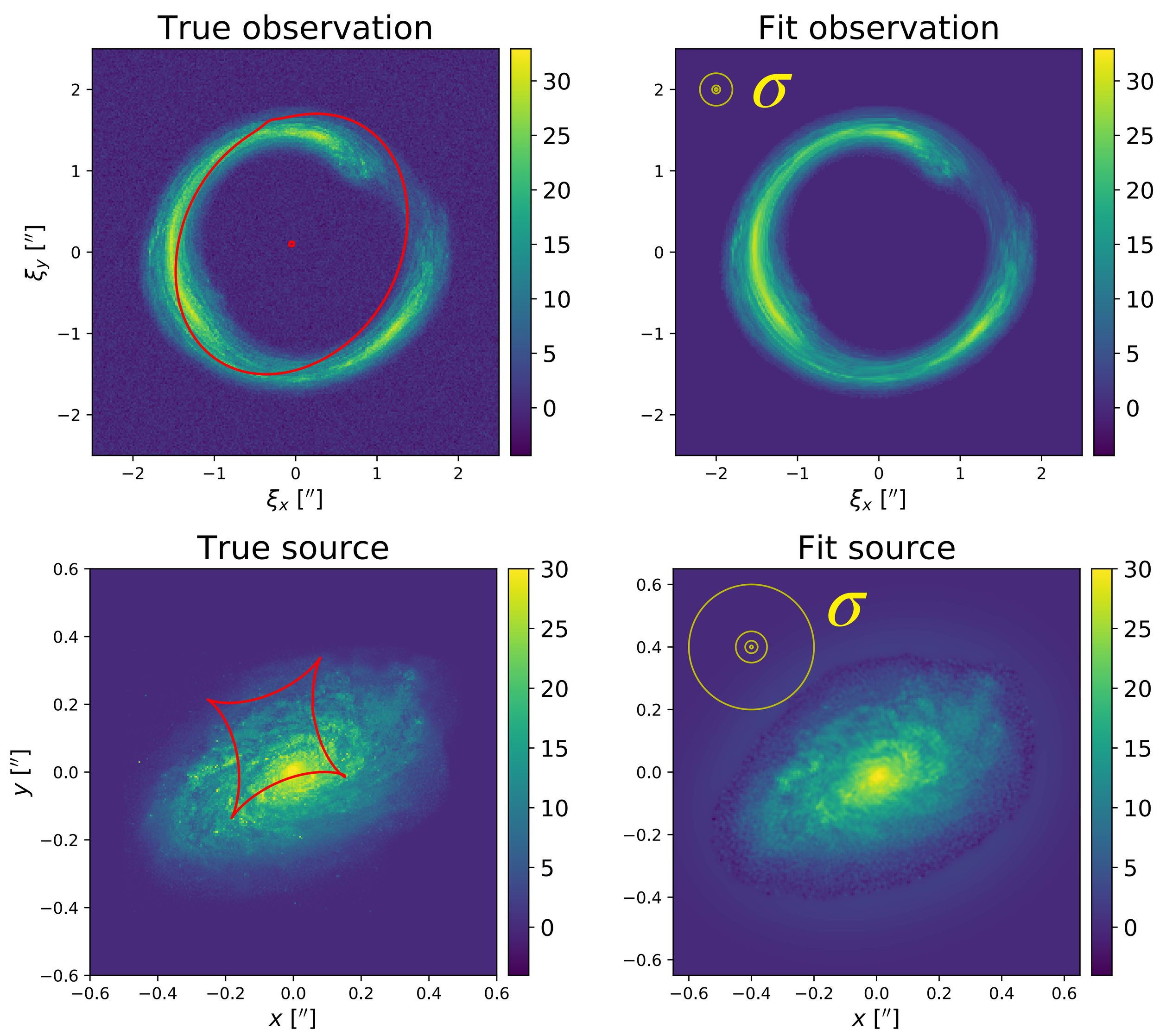
- **Aim:** posteriors for substructure parameters $\mathbf{z}_d$ marginalized over source and lens parameters
- **Neural likelihood-to-evidence ratio estimation:** estimate $p(\mathbf{z}_d \mid \mathbf{x}) / p(\mathbf{z}_d)$
- Train classifier $d(\mathbf{x}, \mathbf{z}_d)$ to distinguish two classes:

  1. $\mathbf{x}, \mathbf{z}_d \sim p(\mathbf{x}, \mathbf{z}_d)$ *data & parameters sampled from joint distribution*
  2. $\mathbf{x}, \mathbf{z}_d \sim p(\mathbf{x})\, p(\mathbf{z}_d)$ *sampled independently*

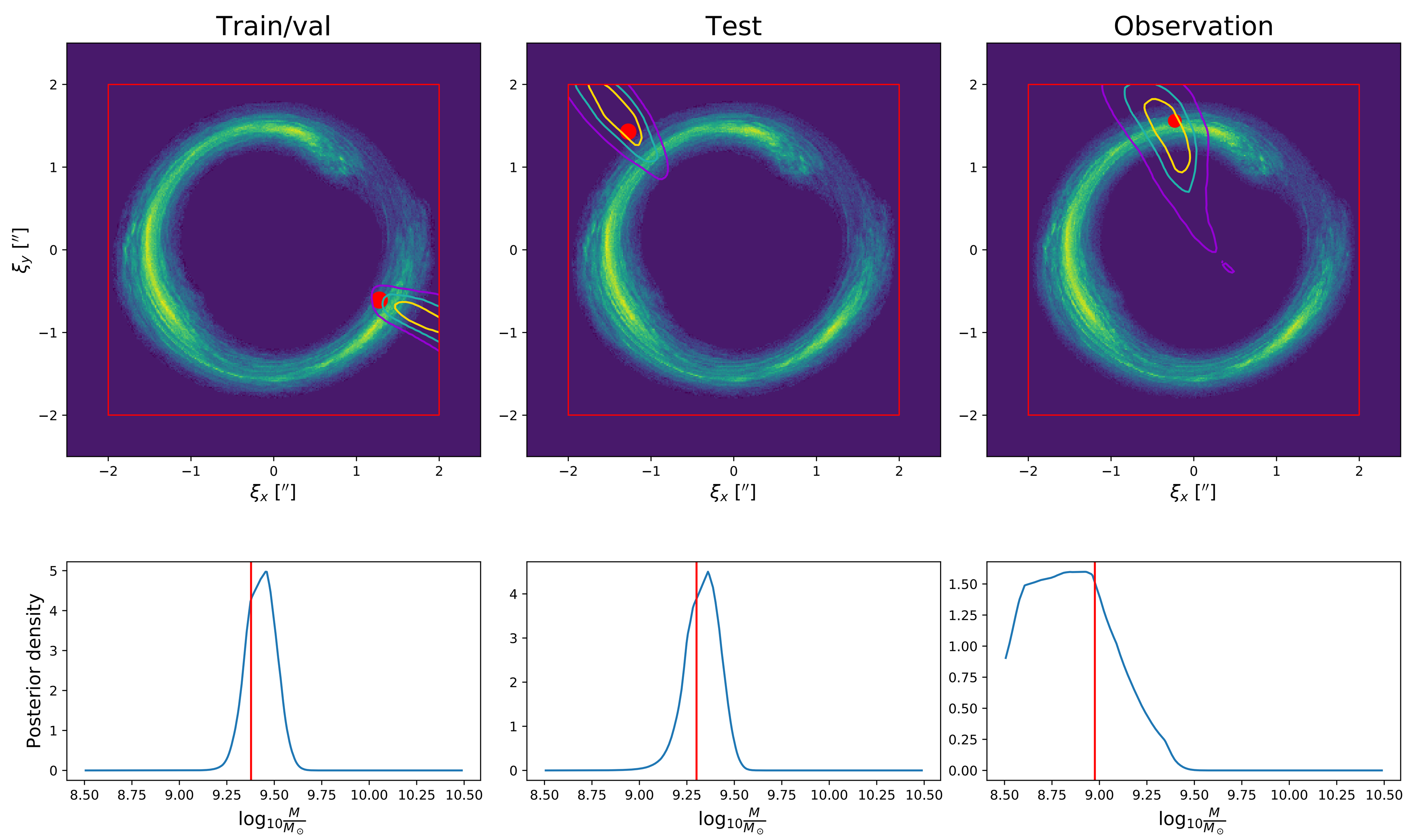
➡ Can recover ratio through $d(\mathbf{x}, \mathbf{z}_d) / (1 - d(\mathbf{x}, \mathbf{z}_d))$

- **Training samples: draw from approximate posterior**
- Implemented using `swyft` (*see* paper *& poster at this workshop!*)
- Mock data analysis of high-resolution image: only 10,000 training samples, runs in a few hours on single GPU. Promising results!
- **Next up:** apply to real Hubble Space Telescope data

Mock analysis: fit results



Mock analysis: subhalo inference results



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