Probabilistic neural network-based reduced-order surrogate for fluid flows
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Introduction

- Neural networks (NNs) have shown their great potential as an universal approximator in physical sciences
- NNs are usually handled as Black box
- No feedback for us
- $L_2$ error based deterministic regression: no notion of confidence intervals
- How is the probabilistic view?
- Variational inference[1], Gaussian process approximation[2]
- Enables us to not only assess model and data but also quantify uncertainty

Probabilistic neural network (PNN)[3]

- Focus on probability distribution of estimation, which can be approximated as linear superposition
- Attempts to get the probability distribution of output directly
- Loss function: Maximization of log-likelihood
- Suitable to utilize the full distribution of estimation by PNN

PNN-based reduced order model with proper orthogonal decomposition

1. Take POD for flow field and obtain temporal coefficients

$$q = q_0 + \sum_{i=1}^{M} a_i \varphi_i$$

2. PNN attempts to predict a temporal evolution of POD coefficients over $n$ time steps from local sensor information $s$ at the first time step

$$s(s^1), \mu(s^1), \sigma(s^1), \pi(s^1)$$

$$\pi(s^1), \mu(s^1), \sigma(s^1)$$

$$p(\alpha(s^1)) = \sum_{i=1}^{M} \pi_i(s^1)N(\mu_i(s^1), \sigma_i(s^1))$$

$$\alpha_{i} = [\alpha_1^i, \alpha_2^i, ..., \alpha_n^i], \text{ where } \alpha^i = [\alpha_1^i, \alpha_2^i, ..., \alpha_n^i]$$

Results

Comparison with benchmark linear method (Gappy POD)

Example: Two-dimensional inviscid shallow water equations

Comparison with benchmark linear method (Gappy POD)

- PNN exhibits the significant advantage even at $n_{\text{sensor}} = 3800$

Conclusions

- Introduced PNN to quantify uncertainties for fluid flow surrogate modeling and data reconstruction
- Provided confidence intervals can be useful for additional sensor placements in fluid flow data recovery tasks[5]

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