CNN-AE/LSTM based turbulent flow forecast on low-dimensional latent space

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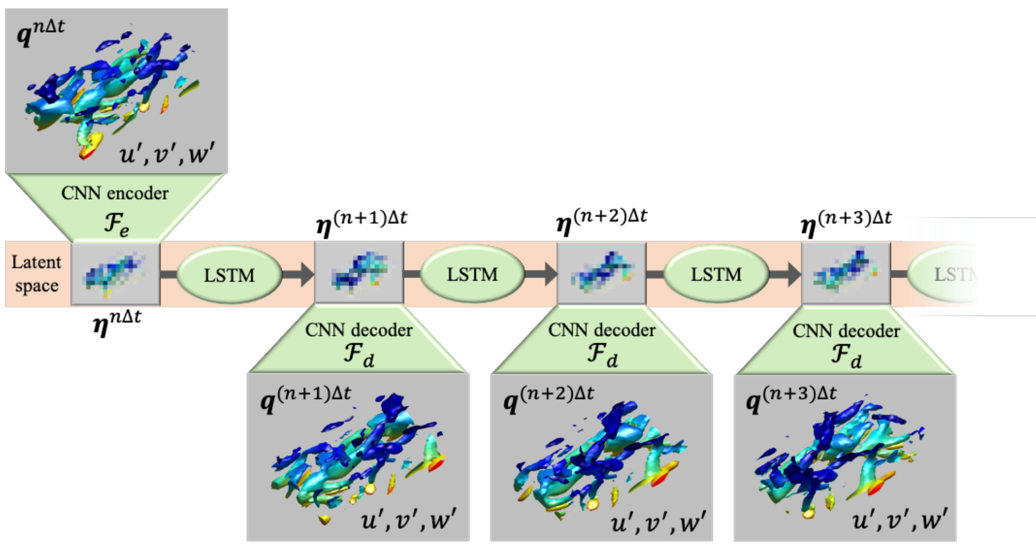

Figure 1: Schematic of the present reduced order model.

Abstract

We propose an unified model of convolutional neural network autoencoder (CNN-AE) and long short-term memory as a reduced order surrogate of high-dimensional nonlinear dynamical systems. The CNN-AE first works as a mapping function for spatial high-dimensional data to a low-dimensional latent space. The LSTM is then utilized to predict a temporal evolution of the latent vectors obtained by the CNN-AE. Combining them, the spatio-temporal high-dimensional system can be represented by only tracking the temporal evolution of the low-dimensional latent dynamics. As an example of complex high-dimensional data, we consider a turbulent channel flow. The fields reproduced by the present model show statistical agreement with the reference data in time-ensemble sense, which can also be found through an orbit-based analysis.

1 Introduction

Recent development of computational power enables us to analyze various nonlinear systems, which are generally represented by high-dimensional data with the immense number of discretization points in both space and time directions. However, this trend also suggests that we should start to care

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about the expansion of computational storage and resources to handle the problem with the immense dimension. From this view, a neural network based reduced order surrogate has recently been attracting an attention as an effective tool to deal with high-dimensional nonlinear dynamical systems. For instance, Maulik et al. [1] proposed a reduced order model constructed by a convolutional neural network autoencoder (CNN-AE) and a long short-term memory (LSTM). The CNN-AE was used to reduce a spatial dimension of Burgers equation, and then the LSTM was employed to integrate the temporal evolution of the reduced vectors by the CNN-AE. While the advection-dominated system was hard to represent on low-dimensional space using proper orthogonal decomposition (POD) and Galerkin projection, their model can represent the field with only two latent-space dimensions. The similar idea was extended to a higher dimension problem by Hasegawa et al. [2]. They applied the CNN-AE and the LSTM to two-dimensional flows around bluff bodies whose shape are randomly defined, and demonstrated that their reduced order surrogate is able to reproduce flow fields for unseen-shaped bluff bodies, which suggests the model learns not just the flow fields used for training but the physics of the dynamical system. They also examined the Reynolds number dependence of their reduced order modeling [3]. Moreover, Carlberg et al. [4] has recently utilized the AE and POD to reduce dimension of high-dimensional flows, and predicted the temporal evolution of the low-dimensional state using various methods, e.g, dynamic mode decomposition, support vector regression, sparse identification of nonlinear dynamics, and so on. They reported that the AE can reduce the dimension of turbulent wake over an extruded half-cylinder at $\mathrm{Re} = 350$ from 320 to only 24. As can be seen above, the neural network based reduced order surrogate works effectively in handling high-dimensional nonlinear systems; however, there are few studies where the surrogate model is applied to more practical fields, which require a lot of spatio-temporal modes to reconstruct the energetic systems. Motivated by the studies above, we here consider a turbulent channel flow as an example of practical flow field in order to examine the applicability of CNN-AE/LSTM based reduced order surrogate. Let us note that the present investigation is challenging because it is widely known that the low-dimensionalization for the turbulent channel flow can be regarded as a conundrum — for example, the channel flow at friction Reynolds number $\mathrm{Re}_\tau = 180$ requires 7260 spatial POD modes to reconstruct 95% of energy [5].

2 CNN-AE/LSTM based reduced order model

The present model consists of the CNN-AE model and the LSTM model, as illustrated in figure 1. The initial flow velocity field $\boldsymbol{q}^{n\Delta t}$ obtained by the reference simulation is fed into the trained CNN encoder $\mathcal{F}_e$ so that we can obtain a low-dimensional latent vector $\boldsymbol{\eta}^{n\Delta t}$ which retains the information of input data on the high-dimensional space. The LSTM model $\mathcal{F}_L$ then predicts the latent vector at the next time step $\boldsymbol{\eta}^{(n+1)\Delta t}$ by feeding that at the current time steps $\boldsymbol{\eta}^{n\Delta t}$. The output of LSTM model is recursively utilized as the input for the next time steps to obtain the temporal evolution of the encoded fields without recovering to original dimension, which enables us to achieve lower computational burden than the original solver. The flow field on the high dimensional space can be reconstructed by the trained CNN decoder $\mathcal{F}_d$ with the predicted latent fields by the LSTM. Summarizing above, the present approach can be mathematically expressed as,

$$\boldsymbol{\eta}^{n\Delta t} = \mathcal{F}_e(\boldsymbol{q}^{n\Delta t}), \quad \boldsymbol{\eta}^{(n+1)\Delta t} = \mathcal{F}_L(\boldsymbol{\eta}^{n\Delta t}), \quad \boldsymbol{q}^{(n+1)\Delta t} \approx \mathcal{F}_d(\boldsymbol{\eta}^{(n+1)\Delta t}). \tag{1}$$

Although the details will be offered later, the time step of LSTM Δt_{ML} is set to wider than that of a reference simulation Δt_{Ref} so that the model can have a significant advantage in terms of the Courant-Friedrichs-Lewy (CFL) constraint which restricts the width of computational time steps for stable simulation process. Also using this setup, it can be expected that the present model achieves the low computational cost from the perspectives on both spatial dimension and temporal integration.

3 Application to turbulent flows

In this work, we apply the present reduced-order model to a turbulent channel flow at $\mathrm{Re}_\tau = 110$. The training and test data are prepared with direct numerical simulation (DNS) by numerically solving incompressible continuity and Navier–Stokes equations. The size of the computational domain and the number of grid points here are $(L_x, L_y, L_z) = (\pi\delta, 2\delta, 0.5\pi\delta)$ and $(N_x, N_y, N_z) = (32, 64, 32)$, where δ represents channel half-width. The time step in the simulation corresponds to $\Delta t^+_{\mathrm{DNS}} = 0.0385$, while the time interval for training data is $\Delta t^+_{\mathrm{ML}} = 3.85$. Note again that this setting for

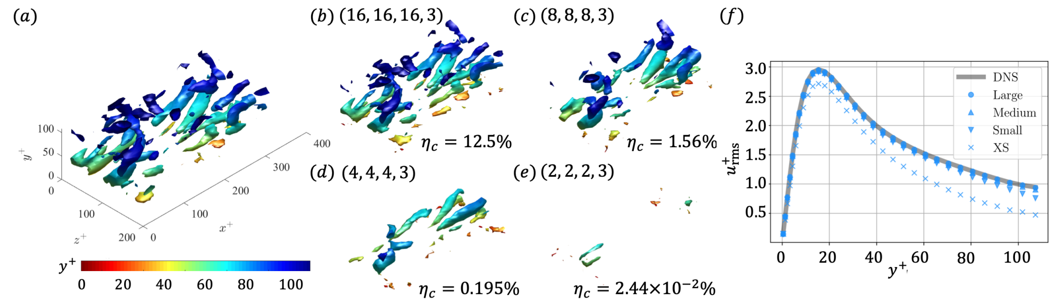

Figure 2: Isosurfaces of the second invariant of velocity gradient tensor ($Q^+ = 0.01$) of (a) DNS, CNN-AE with (b) large, (c) medium, (d) small, and (e) extra small latent spaces. The size of latent vector and compression ratio against the original data are also shown. (f) Root mean squared value of u'.

computational time steps enables the model to have a significant advantage against conventional time-integration solvers since the neural network-based model is not suffered from the CFL constraint. The input attribute is the velocity fluctuations, $\{u', v', w'\}$.

Let us first examine the ability of CNN-AE model to check how low-dimensional space can go well while keeping information of high-dimensional dynamics. We here consider four CNN-AE models, as shown in figure 2. The number above each figure is the size of latent vector, and η_c is the ratio of size of latent space to the original dimension. For the training of CNN-AE, we use 10000 snapshots over 38500 viscous time. As shown in figures 2 (a)-(c), fine structures can be reconstructed with the large and medium models, which implies that the machine learning models can successfully low-dimensionalize the flow field. This can also be seen from the root-mean-squared value of u' in figure 2 (f). The curves obtained by the large and medium model are in reasonable agreement with the reference DNS. The small model can also represent large-scale structures, although the smaller structure is discarded, as presented in figure 2 (d). This is likely because the CNN-AE with the small-size latent vector contains more number of pooling layers than that inside the large or medium model to achieve lower dimensionalization. This enforces the lack of information for small-scale structures through the repeated pooling operations. This trend due to the overcompression can also be found in the extra small model.

Next, we construct the LSTM model to predict the temporal evolution of latent vectors. For the spatial low-dimensionalization, we use the medium CNN-AE model which can capture the trends of DNS as discussed above. Analogous to the CNN-AE training, we use 10000 realizations obtained by reference DNS data with the CNN-AE for the LSTM training. The representative instantaneous flow field produced by the present reduced-order model is visualized using the Q-criterion isosurface, in figure 3 (b). The produced vortical structure shows the similar trend to that in the DNS field. In addition, to check the long-term behavior produced by the present model, we also assess the three-dimensional

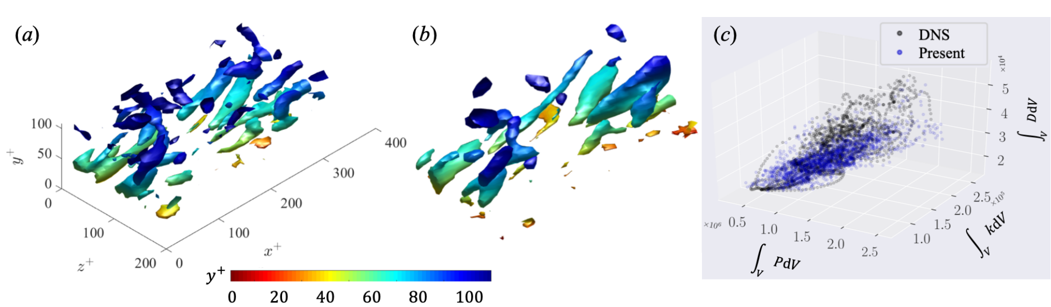

Figure 3: Isosurfaces of the second invariant of velocity gradient tensor ($Q^+ = 0.01$) of (a) DNS and (b) the present model. (c) Turbulence statistics-based orbits of the reference DNS and the present model.

orbit of turbulent kinetic energy (TKE) $\int_V kdV$, production of TKE $\int_V PdV$, and dissipation of TKE $\int_V DdV$, as presented in figure 3 (c). As shown here, the orbit of the present model overlaps that of the reference, which implies that the attractor may exist in the similar position on the orbit space. These results indicate that the present reduced-order model can successfully represent the spatio-temporal chaotic dynamics of turbulent channel flow despite that the low-dimensional latent vector is only followed by the LSTM, although it does not match with the reference at the instantaneous sense. Note that the present LSTM model can integrate the system over 38500 viscous time taking 5.74 seconds on the GPU (NVIDIA Tesla V100), while the reference simulation spends 193 seconds on the CPU (Xeon Gold 6130, 2.1 GHz) for the same time range. In other words, the computational benefit can also be received with the use of proposed approach while achieving the reasonable accuracy as discussed above.

4 Conclusions

We proposed an unified model of a convolutional neural network autoencoder (CNN-AE) and a long short-term memory (LSTM) as a reduced order surrogate of high-dimensional dynamical systems. The CNN-AE mapped a high-dimensional data into a low-dimensional latent space, and the LSTM was then utilized to predict a temporal evolution of the latent vector. As an example of complex high-dimensional systems, a turbulent channel flow was considered. The flow fields generated by the present reduced order model were in statistical agreement with the reference DNS data in time-ensemble sense, which can also be found with an orbit-based analysis. Although we use a regular three-dimensional convolutional neural network for spatial low-dimensionalization, it would be possible to reduce dimension more efficiently by arranging the customized model — for instance, Fukami et al. have recently proposed a hiearchical autoencoder to achieve more efficient low-dimensionalization than conventional autoencoders and POD [6]. We strongly believe that our proposal and investigation can be one of clues toward more practical application of neural-network based reduced order surrogate for complex high-dimensional systems.

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Broader impact

There are no negative impacts associated with this work. The positive influence is that we can reduce a computational cost and storage by using the dimension reduction technique suggested in this work. Since our proposal predicts the temporal evolution of dynamics in the low-dimensional space and the time step is not suffer from CFL constraint, we can further save the computational cost in any problem settings by using the present CNN-AE/LSTM based framework. Moreover, it can also be expected that the CNN-AE based dimensional reduction technique will be able to tackle the problem of data explosion. Users can keep *big data* in science field with little storage by mapping data into latent space using CNN-AE.