
Dispersive qubit readout with machine learning

Enrico Rinaldi

University of Michigan, Ann Arbor MI, USA
and Theoretical Quantum Physics Laboratory, RIKEN, Japan
and Center for Quantum Computing, RIKEN, Japan
erinaldi.work@gmail.com

Roberto Di Candia

Department of Communications and Networking, Aalto University, Finland
rob.dicandia@gmail.com

Simone Felicetti

Istituto di Fotonica e Nanotecnologie, Consiglio Nazionale delle Ricerche (IFN-CNR), Italy
felicetti.simone@gmail.com

Fabrizio Minganti

Institute of Physics, École Polytechnique Fédérale de Lausanne (EPFL), Switzerland
and Theoretical Quantum Physics Laboratory, RIKEN, Japan
fabrizio.minganti@gmail.com

Abstract

Open quantum systems can undergo dissipative phase transitions, and their critical behavior can be exploited in sensing applications. For example, it can be used to enhance the fidelity of superconducting qubit readout measurements, a central problem toward the creation of reliable quantum hardware. A recently introduced measurement protocol, named “critical parametric quantum sensing”, uses the parametric (two-photon driven) Kerr resonator’s driven-dissipative phase transition to reach single-qubit detection fidelity of 99.9% [arXiv:2107.04503]. In this work, we improve upon the previous protocol by using machine learning-based classification algorithms to *efficiently and rapidly* extract information from this critical dynamics, which has so far been neglected to focus only on stationary properties. These classification algorithms are applied to the time series data of weak quantum measurements (homodyne detection) of a circuit-QED implementation of the Kerr resonator coupled to a superconducting qubit. This demonstrates how machine learning methods enable a faster and more reliable measurement protocol in critical open quantum systems.

1 Introduction

A sensing device is a system (weakly) coupled to a second system, the target, characterized by an unknown parameter. By observing the response of the sensor to the coupling, one can estimate the value of the parameter of the target [1]. The larger the response of the sensor, the better the estimation of the unknown parameter. For this reason, quantum systems around criticality have been proposed as sensors, because the diverging susceptibility characterizing second-order phase transition empowers precise parameter estimation. Such a metrological advantage of quantum phase transitions, however, is limited by a practical consideration: a diverging susceptibility is associated

with a critical slowing down, i.e., the emergence of an infinitely-long timescale. Thus, the price to pay for increased precision is a diverging measurement time [2]. Similarly to thermal and quantum phase transitions, an open quantum system can develop second-order Dissipative Phase Transitions (DPTs) [3, 4] associated with a diverging susceptibility [5]. Very little has been done in investigating the metrological properties of DPTs [6, 7].

Two-photon Kerr resonators are remarkably simple open quantum systems that are at the center of intense experimental research in quantum optics and information [8, 9]. The two-photon Kerr resonator undergoes a second-order DPT [10, 11] where the photon number $\langle \hat{a}^\dagger \hat{a} \rangle$ has a non-analytical change. In Ref. [12], the authors introduced a quantum measurement protocol called “critical parametric quantum sensing” using the two-photon Kerr resonator. Using this DPT, the state of a qubit (two-level quantum system) coupled to the sensor can be inferred with very high precision. Indeed, if the qubit is in the state $|\uparrow\rangle$, after a quick transient dynamics, the resonator is almost empty, while if the state is $|\downarrow\rangle$ the resonator contains many photons. In a first approximation, a homodyne measurement of the photon field allows assessing the state of a qubit with extremely high fidelity (up to 99.9%).

This protocol, however, relies on a measurement of the long-time (stationary) properties of the Kerr resonator, thus ignoring all the information coming from the short-time dynamics of the system. From a physical point of view, the importance of short-time measurement is clear when considering both non-ideal dispersive measurement and including qubit dissipation and decoherence (both sources of *noise* in current NISQ-era quantum hardware). Thus, in realistic systems, a measurement protocol must be performed within the qubit coherence time. More generally, growing efforts are dedicated to the development of sensing protocols based on dynamical properties of critical quantum systems [13–17].

From a theoretical point of view, characterizing the *average* dynamics of an open quantum system requires nontrivial computations of the spectral properties of the generator of the dissipative dynamics [18], making it a formidable challenge. This is even more true when discussing individual trajectories, which describe the outcomes of single experimental realizations. Indeed, the complexity of the stochastic noise induces nontrivial correlation properties in the dynamics that cannot be captured by the average dynamics of a Lindblad master equation [19–21]. For this reason, an efficient *dynamic* critical parametric quantum sensor cannot rely on simple estimation strategies based on average properties of the system. On the other hand, machine learning algorithms are well suited for such a task and are known to work well even in the presence of experimental noise [22].

In this paper, we present a novel machine learning approach to critical parametric quantum sensing that improves the protocol metrological power by taking into account the dynamical aspect of the DPT. The motivation for this improvement is the following. Even though the protocol using the steady state reaches high fidelity, it is based on the assumption of a perfect qubit, and as such it requires waiting for a sufficiently long time before the Kerr resonator reaches its steady state. In actual experimental realizations, however, time is a resource, and the shorter a measurement takes, the less the qubit is prone to errors. The procedure we propose is a machine learning-based classification of measurement time series, allowing to extract information on the qubit state from the quantum fluctuations of the Kerr resonator dynamics, even at short times.

2 Two-photon Kerr resonator

The Hamiltonian of a two-photon driven Kerr-resonator coupled to a qubit is

$$\hat{H}_{\text{Kerr}} = \omega \hat{a}^\dagger \hat{a} + \omega_q \hat{\sigma}^+ \hat{\sigma}^- + \frac{\epsilon}{2} (\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2} \hat{a}^2 + g (\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+), \quad (1)$$

where $\hat{a}(\hat{a}^\dagger)$ is the bosonic annihilation(creation) operator and $\hat{\sigma}^-(\hat{\sigma}^+)$ is the lowering(raising) spin operator. This model can be realized in various photonic platforms, such as circuit-QED implementation, where a driven resonator is coupled with a superconducting quantum interference device (SQUID) element [23, 24]. We define the pump-resonator detuning ω , the effective pump-power ϵ , and the SQUID-induced non-linearity χ .

Photons will continuously escape the Kerr resonator, and the number of photons emitted is directly proportional to the number of photons in the Kerr resonator. This is described by the Lindblad master

equation for the system density matrix [1]

$$\frac{\partial}{\partial t} \hat{\rho}(t) = -i[\hat{H}, \hat{\rho}(t)] + \Gamma[2\hat{a}\hat{\rho}(t)\hat{a}^\dagger - \{\hat{a}^\dagger\hat{a}, \hat{\rho}(t)\}] \quad (2)$$

where $\Gamma \geq 0$ is the dissipation rate, and it is used as a characteristic timescale.

Dispersive-readout protocols assume the qubit-resonator coupling to be in the linear dispersive regime, where the qubit-resonator detuning $\Delta = |\omega_q - \omega_r| \gg |\omega_q + \omega_r|/2$. In such a regime, the passage of excitation between the qubit and the resonator is suppressed, and the overall effect of the qubit on the resonator is to introduce an effective shift on the frequency of the resonator by $\delta\omega = g^2/\Delta$. Indeed, if the qubit is $|\uparrow\rangle$ ($|\downarrow\rangle$), a photon inside the resonator has energy $\omega + \delta\omega/2$ ($\omega - \delta\omega/2$). In this dispersive regime, a transition occurs at a critical frequency, such that $\omega = \sqrt{\epsilon^2 - \Gamma^2}$.¹ As such, the small change $\delta\omega$ can determine if the system is in the phase with a few photon or in that with many photons, as shown in Fig. 1, where on the left we show the photon number inside the cavity and on the right we plot the qubit state (see also the discussion in [12]). Consequently, by collecting the number of photon emitted, we can determine the state of the qubit.

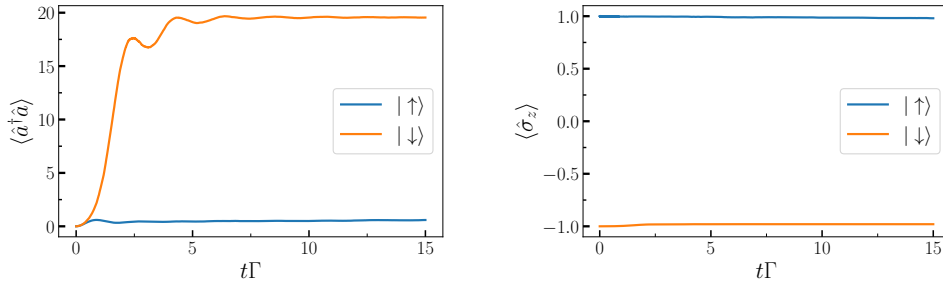


Figure 1: The number of photons and the qubit state averaged over 1000 quantum trajectories for the two different initial qubit states at $\epsilon = 1.67$ and $\delta\omega = 2.3$. The steady-state solution is reached around $t\Gamma \approx 6$.

There is, however, another unwanted effect stemming from the coupling to the Kerr resonator. Indeed, the Kerr resonator induces an effective dissipation on the qubit, introducing additional unwanted errors in the measurement. Since the probability that a quantum jump induced by the qubit-resonator coupling increases with time, the shorter the required measurement time, the better the outcome. For example, in Fig. 2 we show that the qubit flips due to the coupling to the cavity. Therefore, if we are to consider only the long-time dynamics, in both cases we would wrongly guess the initial state of the qubit. This is the first reason to consider the Kerr resonator dynamics at short time.

Furthermore, from a metrological point of view, the main theoretical reason for this work is the fact that, contrary to quantum phase transitions, the critical slowing down of DPTs only affects certain observables. For instance, at the critical point of the two-photon Kerr resonator, the critical slowing down only affects $\langle \hat{a} \rangle$, but leaves unaffected $\langle \hat{a}^\dagger \hat{a} \rangle$. This is a common feature of DPTs, and it was demonstrated also for the XYZ model [20]. Since the protocol proposed in Ref. [12] uses photon number to estimate, e.g., the state of a qubit, *it is possible to obtain metrological advantages from the diverging susceptibility of a DPT avoiding the critical slowing down*. Consequently, the system should display critical fluctuations even at short times, allowing to extract information on the qubit state from the emitted field of the resonator.

3 Methods

These critical fluctuations can be clearly seen in actual experimental realizations (or their numerical simulations) as shown in Fig. 2 at short time. By comparing this single-shot dynamics in the left panel, we see important fluctuations of the Kerr-resonator state, far larger than those in the averaged dynamics (for instance, a small peak around $t\Gamma = 1$ when the qubit is initialized in $|\downarrow\rangle = |0\rangle$). This

¹To be precise, the DPT occurs in the “thermodynamic” limit $\chi \rightarrow 0$ [11]. Nevertheless, for small χ , a sharp crossover occurs.

information, however, can be quite challenging to extract from an analytical point of view, because it would require to obtain the Liouvillian superoperator eigenvalues and eigenmatrices (i.e., the spectral decomposition of the generator of the dynamics) [11]. For this reason, we will employ a machine learning algorithm to guess the qubit state exploiting these dynamical critical fluctuations.

Since we are interested in simulating actual experimental measurement outcomes, we resort to homodyne quantum trajectories. Instead of the average dynamics of the system in Eq. (2), we simulate a set of stochastic Schrödinger equations where noise is sampled at each time step [1]. For a fixed set of parameters in \hat{H}_{Kerr} we simulate 1000 quantum trajectories of the system starting from the two different states of the qubit $|0\rangle = |\downarrow\rangle$ and $|1\rangle = |\uparrow\rangle$ using QuTiP [25, 26]. We save the homodyne quantum evolution for several observables related to the resonator (sensor), such as the number of photons $\langle \hat{a}^\dagger \hat{a} \rangle$, and to the qubit (target), such as $\langle \hat{\sigma}_z \rangle$.

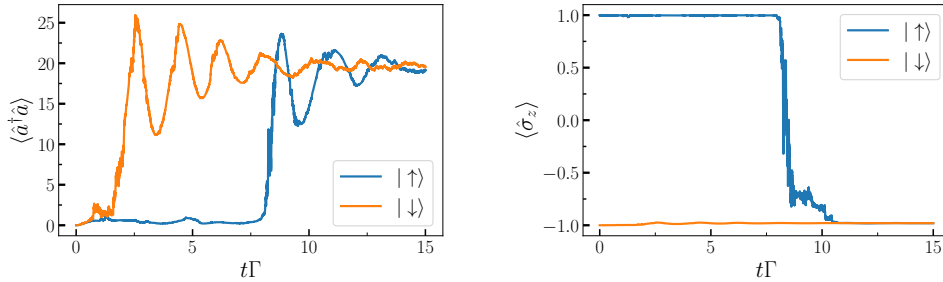


Figure 2: The number of photons and the qubit state for a single quantum trajectory where one of the qubit states flips (fixed $\epsilon = 1.67$ and $\delta\omega = 2.3$). A measurement protocol based on the long-time (stationary) properties would fail to recognize that one qubit was in the $|\uparrow\rangle$ state. Machine learning classifiers can be accurate even with short-time data, when qubits have not had enough time to be altered by the environment.

In circuit-QED experiments, the parameter Γ is fixed by the manufacturing process, the non-linearity χ is set by the SQUID characteristics, while the detuning Δ and the drive intensity ϵ can be easily tuned in real time. We choose to save data with a maximal time resolution of $\Delta t = 10^{-3}/\Gamma$ and up to $t\Gamma = 15$, corresponding to state-of-the-art homodyne measurement frequencies. The other relevant time scales, in units of Γ , are the total time of measurement t_f and the instrument measurement smoothing scale τ , corresponding to a time interval over which individual values can not be obtained and are averaged over by the measurement instrument. These two time parameters can be optimized by the protocol to achieve the highest fidelity.

We choose to analyze the quantum trajectories of the observable \hat{x} using two simple machine learning classifiers for time series. Both classifiers employ a Support Vector Machine algorithm (SVC) with radial basis function kernels (rbf) to discover non-linearity in the feature space. The first classifier (named TAB+SVC(rbf)) is applied to a feature space identified with all the time points of the trajectory, therefore neglecting the time ordering and time correlations. On the other hand, the second classifier (named RIFE+SVC(rbf)) acts on a meta-feature space which is constructed from random interval features (RIFE) [27], such as the average value or the slope of a random time interval of the trajectory. To estimate the error on the classification accuracy we utilize repeated cross-validation folds, with 100 repetitions and 5 folds.

Our choice of classifiers is limited and dictated by simplicity. Similar results should hold irrespective of the classifier algorithm as long as the time series dynamics can be captured. Ideally, classifiers requiring less data to achieve high accuracy would be preferred because that would improve the experimental setup. Higher accuracy can be achieved by including careful feature engineering and feature selection steps, which we have left for future investigations.

4 Results

We want to tune the Kerr resonator to be the best possible measurement instrument. In Candia et al. [12], such an optimal point was inferred from analytical considerations on the steady state. Here

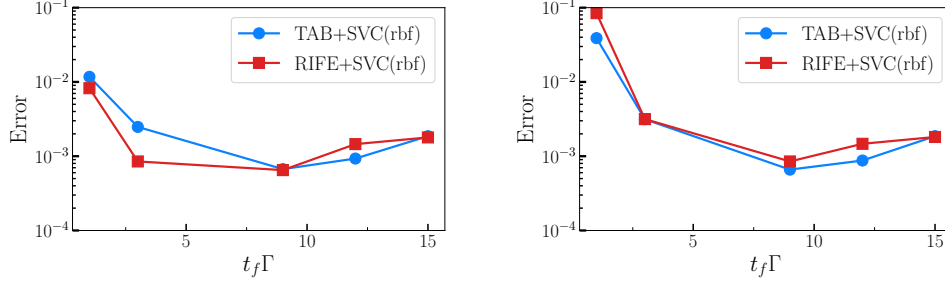


Figure 3: The error (1-accuracy) made by different classifiers as a function of the measurement time $t_f\Gamma$ (fixed $\epsilon = 1.67$ and $\delta\omega = 2.3$). The left panel is for $\tau\Gamma = 10^{-3}$ and the right panel for $\tau\Gamma = 10^{-1}$.

instead we consider the dynamical properties. We scanned the parameter set $\{\epsilon, \omega\}$ (which can be easily controlled in standard experimental implementations) and verified the accuracy provided by our choice of classifiers for the state of a qubit initialized either in $|\uparrow\rangle$ or $|\downarrow\rangle$, where the accuracy is the ratio between the number of correct results over the total number of simulations (the variability of the accuracy in our cross-validation procedure is always of the order of 0.002 or less). This procedure is repeated for different total measurement time $t_f\Gamma$ and for different measurement smoothing scales $\tau\Gamma$: note that this procedure amounts to changing the quality and quantity of samples used in the supervised learning classification task.

In Fig. 3, we plot the results obtained at various $t_f\Gamma$. In the left panel, where $\tau = 10^{-3}/\Gamma$, we see that for very short times the algorithm is capable of determining the qubit state in a very short time $t_f\Gamma = 2$ and very high accuracy (errors $\simeq 1 - 9 \times 10^{-4}$). If we increase $\tau = 10^{-1}/\Gamma$, we can obtain similar accuracy ($\simeq 1 \times 10^{-3}$), but we need to wait for longer times ($t_f\Gamma = 9$).

These results demonstrate that, exploiting critical fluctuations, it is possible to obtain a performing measurement instrument, capable of determining the state of a qubit with extreme high precision and in very short time. In the future, we plan to further investigate the properties of critical open quantum system on a more mathematical ground, and to test this protocol on actual experimental data, including measurement noise and qubit dissipation effects.

Acknowledgments and Disclosure of Funding

We acknowledge the use of computing time on the RIKEN Hokusai BigWaterfall cluster. E. R. is supported by Nippon Telegraph and Telephone Corporation (NTT) Research. R. D. acknowledges support from the Marie Skłodowska Curie fellowship number 891517 (MSC-IF GreenMIQUEC).

References

- [1] H.M. Wiseman and G.J. Milburn. Quantum measurement and control. *Cambridge University Press (Cambridge)*, 2010. doi: 10.1017/CBO9780511813948. URL <https://doi.org/10.1017/CBO9780511813948>.
- [2] Marek M. Rams, Piotr Sierant, Omyoti Dutta, Paweł Horodecki, and Jakub Zakrzewski. At the limits of criticality-based quantum metrology: Apparent super-heisenberg scaling revisited. *Phys. Rev. X*, 8:021022, Apr 2018. doi: 10.1103/PhysRevX.8.021022. URL <https://link.aps.org/doi/10.1103/PhysRevX.8.021022>.
- [3] J. Jin, A. Biella, O. Viyuela, L. Mazza, J. Keeling, R. Fazio, and D. Rossini. Cluster mean-field approach to the steady-state phase diagram of dissipative spin systems. *Phys. Rev. X*, 6:031011, Jul 2016. URL <http://link.aps.org/doi/10.1103/PhysRevX.6.031011>.

- [4] Riccardo Rota, Fabrizio Minganti, Cristiano Ciuti, and Vincenzo Savona. Quantum critical regime in a quadratically driven nonlinear photonic lattice. *Phys. Rev. Lett.*, 122:110405, Mar 2019. URL <https://link.aps.org/doi/10.1103/PhysRevLett.122.110405>.
- [5] R. Rota, F. Storme, N. Bartolo, R. Fazio, and C. Ciuti. Critical behavior of dissipative two-dimensional spin lattices. *Phys. Rev. B*, 95:134431, Apr 2017. URL <https://link.aps.org/doi/10.1103/PhysRevB.95.134431>.
- [6] Toni L. Heugel, Matteo Biondi, Oded Zilberberg, and R. Chitra. Quantum transducer using a parametric driven-dissipative phase transition. *Phys. Rev. Lett.*, 123:173601, Oct 2019. doi: 10.1103/PhysRevLett.123.173601. URL <https://link.aps.org/doi/10.1103/PhysRevLett.123.173601>.
- [7] Louis Garbe, Matteo Bina, Arne Keller, Matteo G. A. Paris, and Simone Felicetti. Critical quantum metrology with a finite-component quantum phase transition. *Phys. Rev. Lett.*, 124:120504, Mar 2020. doi: 10.1103/PhysRevLett.124.120504. URL <https://link.aps.org/doi/10.1103/PhysRevLett.124.120504>.
- [8] Z. Leghtas, S. Touzard, I. M. Pop, A. Kou, B. Vlastakis, A. Petrenko, K. M. Sliwa, A. Narla, S. Shankar, M. J. Hatridge, M. Reagor, L. Frunzio, R. J. Schoelkopf, M. Mirrahimi, and M. H. Devoret. Confining the state of light to a quantum manifold by engineered two-photon loss. *Science*, 347(6224):853–857, 2015. URL <http://dx.doi.org/10.1126/science.aaa2085>.
- [9] Raphaël Lescanne, Marius Villiers, Théau Peronnin, Alain Sarlette, Matthieu Delbecq, Benjamin Huard, Takis Kontos, Mazyar Mirrahimi, and Zaki Leghtas. Exponential suppression of bit-flips in a qubit encoded in an oscillator. *Nature Physics*, 16(5):509–513, 2020. doi: 10.1038/s41567-020-0824-x. URL <https://doi.org/10.1038/s41567-020-0824-x>.
- [10] N. Bartolo, F. Minganti, W. Casteels, and C. Ciuti. Exact steady state of a kerr resonator with one- and two-photon driving and dissipation: Controllable wigner-function multimodality and dissipative phase transitions. *Phys. Rev. A*, 94:033841, Sep 2016. URL <https://link.aps.org/doi/10.1103/PhysRevA.94.033841>.
- [11] Fabrizio Minganti, Alberto Biella, Nicola Bartolo, and Cristiano Ciuti. Spectral theory of liouvillians for dissipative phase transitions. *Phys. Rev. A*, 98:042118, Oct 2018. URL <https://link.aps.org/doi/10.1103/PhysRevA.98.042118>.
- [12] R. Di Candia, F. Minganti, K. V. Petrovnin, G. S. Paraoanu, and S. Felicetti. Critical parametric quantum sensing. *arXiv:2107.04503*, 2021. URL <https://arxiv.org/abs/2107.04503>.
- [13] Mankei Tsang. Quantum transition-edge detectors. *Phys. Rev. A*, 88(2):021801, August 2013. doi: 10.1103/PhysRevA.88.021801. URL <https://link.aps.org/doi/10.1103/PhysRevA.88.021801>.
- [14] Katarzyna Macieszczak, Mădălin Guță, Igor Lesanovsky, and Juan P. Garrahan. Dynamical phase transitions as a resource for quantum enhanced metrology. *Phys. Rev. A*, 93(2):022103, February 2016. doi: 10.1103/PhysRevA.93.022103. URL <https://link.aps.org/doi/10.1103/PhysRevA.93.022103>.
- [15] Yaoming Chu, Shaoliang Zhang, Baiyi Yu, and Jianming Cai. Dynamic framework for criticality-enhanced quantum sensing. *Phys. Rev. Lett.*, 126:010502, Jan 2021. doi: 10.1103/PhysRevLett.126.010502. URL <https://link.aps.org/doi/10.1103/PhysRevLett.126.010502>.
- [16] Karol Gietka, Lewis Ruks, and Thomas Busch. Exponentially enhanced quantum metrology by quenching superradiant light-matter systems. *arXiv:2110.04048*, 2021. URL <https://arxiv.org/abs/2110.04048>.
- [17] Louis Garbe, Obinna Abah, Simone Felicetti, and Ricardo Puebla. Critical quantum metrology with fully-connected models: From heisenberg to kibble-zurek scaling. *arXiv:2110.04144*, 2021. URL <https://arxiv.org/abs/2110.04144>.
- [18] Daniel A. Lidar. Lecture notes on the theory of open quantum systems. *arXiv:1902.00967*, 2020. URL <https://arxiv.org/abs/1902.00967v2>.

- [19] N. Bartolo, F. Minganti, J. Lolli, and C. Ciuti. Homodyne versus photon-counting quantum trajectories for dissipative kerr resonators with two-photon driving. *Eur. Phys. J.: Spec. Top.*, 226(12):2705–2713, Jul 2017. ISSN 1951-6401. URL <https://doi.org/10.1140/epjst/e2016-60385-8>.
- [20] R. Rota, F. Minganti, A. Biella, and C. Ciuti. Dynamical properties of dissipative xyz heisenberg lattices. *New J. Phys.*, 20(4):045003, 2018. URL <http://stacks.iop.org/1367-2630/20/i=4/a=045003>.
- [21] Carlos Sánchez Muñoz, Berislav Buča, Joseph Tindall, Alejandro González-Tudela, Dieter Jaksch, and Diego Porras. Symmetries and conservation laws in quantum trajectories: Dissipative freezing. *Phys. Rev. A*, 100:042113, Oct 2019. doi: 10.1103/PhysRevA.100.042113. URL <https://link.aps.org/doi/10.1103/PhysRevA.100.042113>.
- [22] Élie Genois, Jonathan A. Gross, Agustin Di Paolo, Noah J. Stevenson, Gerwin Koolstra, Akel Hashim, Irfan Siddiqi, and Alexandre Blais. Quantum-tailored machine-learning characterization of a superconducting qubit. *arXiv:2106.13126*, 2021. URL <https://arxiv.org/abs/2106.13126>.
- [23] Philip Krantz, Yarema Reshitnyk, Waltraut Wustmann, Jonas Bylander, Simon Gustavsson, William D Oliver, Timothy Duty, Vitaly Shumeiko, and ner Delsing. Investigation of nonlinear effects in josephson parametric oscillators used in circuit quantum electrodynamics. *New Journal of Physics*, 15(10):105002, oct 2013. doi: 10.1088/1367-2630/15/10/105002. URL <https://doi.org/10.1088/1367-2630/15/10/105002>.
- [24] Z. R. Lin, K. Inomata, K. Koshino, W.D. Oliver, Y. Nakamura, J.S. Tsai, and T. Yamamoto. Josephson parametric phase-locked oscillator and its application to dispersive readout of superconducting qubits. *Nature communications*, 1(4480):4480, 2014. doi: 10.1038/ncomms5480. URL <https://www.nature.com/articles/ncomms5480>.
- [25] Jesper Robert Johansson, Paul D. Nation, and Franco Nori. Qutip: An open-source python framework for the dynamics of open quantum systems. *Computer Physics Communications*, 183(8):1760–1772, 2012. ISSN 0010-4655. doi: <https://doi.org/10.1016/j.cpc.2012.02.021>. URL <https://www.sciencedirect.com/science/article/pii/S0010465512000835>.
- [26] Jesper Robert Johansson, Paul D. Nation, and Franco Nori. Qutip 2: A python framework for the dynamics of open quantum systems. *Computer Physics Communications*, 184(4):1234–1240, 2013. ISSN 0010-4655. doi: <https://doi.org/10.1016/j.cpc.2012.11.019>. URL <https://www.sciencedirect.com/science/article/pii/S0010465512003955>.
- [27] J.J. Rodríguez and C.J. Alonso. Support vector machines of interval-based features for time series classification. *Research and Development in Intelligent Systems XXI, SGAI 2004*, 2005. doi: 10.1007/1-84628-102-4_18. URL https://link.springer.com/chapter/10.1007/1-84628-102-4_18.

Checklist

1. For all authors...
 - (a) Do the main claims made in the abstract and introduction accurately reflect the paper’s contributions and scope? [Yes] See Section 1.
 - (b) Did you describe the limitations of your work? [Yes] See Section 3
 - (c) Did you discuss any potential negative societal impacts of your work? [No] We did not find potential negative societal impacts
 - (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes] We have read the ethics review guidelines
2. If you are including theoretical results...
 - (a) Did you state the full set of assumptions of all theoretical results? [N/A]
 - (b) Did you include complete proofs of all theoretical results? [N/A]
3. If you ran experiments...

- (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [No] The code and data is available for anyone interested
 - (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See Section 4
 - (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] See Section 4
 - (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No]
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
- (a) If your work uses existing assets, did you cite the creators? [N/A]
 - (b) Did you mention the license of the assets? [N/A]
 - (c) Did you include any new assets either in the supplemental material or as a URL? [N/A]

 - (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [N/A]
 - (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [N/A]
5. If you used crowdsourcing or conducted research with human subjects...
- (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [N/A]
 - (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [N/A]
 - (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [N/A]