A probabilistic deep learning model to distinguish cusps and cores in dwarf galaxies

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Abstract

Numerical simulations within a cold dark matter (DM) cosmology predict halos with a characteristic density profile with a logarithmic inner slope of -1. Various methods, such as Jeans and Schwarzschild modelling, have been used in an attempt to determine the inner density of observed dwarf galaxies, in order to test this theoretical prediction. Here, we develop a mixture density convolutional neural networks (MDCNNs) to derive a posterior distribution of the inner density slopes of DM halos. We train the MDCNN on a suite of simulated galaxies from the NIHAO and AURIGA projects, inputting line-of-sight velocities and 2D spatial information of the stars within simulated galaxies. The output of the MDCNN is a probability density function representing the posterior probability of a certain slope to be the correct one, thus producing accurate and complex information on the uncertainty of the predictions. The model recovers accurately the correct inner slope of dwarfs: $\sim$82% of the galaxies have a derived inner slope within $\pm$0.1 of their true value, while $\sim$98% within $\pm$0.3. We then apply our model to four Local Group dwarf spheroidal galaxies and find similar results to those obtained with the Jeans modelling based code GRAVSPHERE.

1 Introduction

The ΛCDM model has produced very satisfactory results in explaining the large-scale structure of the universe and the basic properties of the galaxies in it (Aghanim et al., 2020). However, observations in galactic-scale systems have shown several discrepancies with the model's predictions. In the literature, these problems have traditionally been referred to as the small-scale problems of the ΛCDM model.

One of the main and oldest of these problems is the so-called cusp/core problem. Numerical simulations within the ΛCDM model form dark matter halos (in which galaxies inhabit) with a characteristic mass density profile that has an internal slope on a logarithmic scale of $\gamma \sim -1$ (Navarro et al., 1996). This profile does not agree with observations of some galaxies with well-measured rotation curves, which prefer settings with less steep internal density slopes of $0.5 - 0$ (e.g. Simon et al., 2005; de Blok et al., 2008; Moore, 1994). Traditionally, the inner part of density profiles with flatter internal slopes has been called a core, and the inner part of density profiles with the large slopes predicted by the ΛCDM model has been called a cusp.

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It has been shown that cores can be explained in the ΛCDM paradigm by considering the effect that baryons have on dark matter. If a dwarf galaxy slowly accretes gas and is then suddenly blown away by processes such as stellar winds or supernova explosions, the dark matter expands, decreasing the density of dark matter in the central part of the halo in which the galaxy is located and potentially forming a core if certain reasonable conditions are met. This process has been successfully modelled in hydrodynamical simulations of galaxies that take this baryonic feedback into account (e.g. Governato et al., 2010; Di Cintio et al., 2014; Tollet et al., 2016; Chan et al., 2015).

Even so, there is much controversy in the investigation of the cusp/core problem, mainly due to the difficulty of obtaining precise measurements of the dark matter distribution of the halo of the observed galaxies, being common that the techniques traditionally used to infer density profiles, such as Jeans modelling, do not allow to constrain the value of the internal slope in such a way that the presence of a core can be ruled out or assured (e.g. van der Marel, 1994; Battaglia et al., 2008; Collins et al., 2021; Read et al., 2019).

In this work we present a novel model to determine the internal slope of the density profile of dark matter halos with a robust quantification of the uncertainty, making use of simulation-based inference with a neural network.

2 Methodology

We train our model using a large set of hydrodynamical simulations of dwarf galaxies with haloes with masses from 10^9 to $10^{11.5}$ from the NIHAO (Wang et al., 2015; Dutton et al., 2020) and AURIGA projects (Grand et al., 2017), up to a total of 183 different galaxies. The use of different models and physical parameters used in these simulations allows us to have a wide range of density profiles in galaxies with similar masses, including both cores and cusps.

We make only use of the velocities and positions of the stars found in the simulated dwarf galaxies to construct Probability Density Functions (PDF) of the distribution of these stars in two different 2D spaces:

- The positional {x,y} space, between -2 kpc and 2 kpc in each coordinate, in the reference system where (x,y) = (0,0) is the center of the galaxy, in a 64x64 grid.
- The $\{\hat{R}_{\rm proj}, \hat{v}_{\rm LOS}\}$ phase space, where $\hat{R}_{\rm proj} = \sqrt{x^2+y^2}/R_{hlr}$ is the radial position normalized by the half-light radius R_{hlr} and $\hat{v}_{\rm LOS} = v_{\rm LOS}/P_{98\%}$ is the line-of-sight velocity normalized by the 98% percentile of the absolute value of $v_{\rm LOS}$ of all stars of the sample. $\hat{R}_{\rm proj}$ ranges from 0 to 1, and $\hat{v}_{\rm LOS}$ ranges from -1 to 1, in a 64x64 grid.

We use mixture density convolutional neural networks (MDCNNs) to map the input data composed of the two PDFs described in last section into the posterior distribution of the inner slopes of the DM profiles of the galaxy associated to those two PDFs. We approximate the posterior distribution of the slopes with the sum of two dimensional Gaussian distribution whose parameters are estimated by the neural network. Our model takes as input a two channel image consisting of the PDFs on the $\{\hat{R}_{\rm proj}, \hat{v}_{\rm LOS}\}$ phase space and the {x,y} phase space separately. The images are passed through 2 convolutional sequential layers. The outputs of the two convolutional branches are then concatenated and fed into a 3 layer fully connected network. The final output consists of 6 parameters which parametrize the joint 2D Gaussian posterior.

Our architecture is influenced by the success of a similar approach to determine galaxy cluster masses with dynamical information of their constituents in Ho et al. (2019) and Kodi Ramanah et al. (2021), where it is shown that a CNN applied to a PDF of positions and velocities of the galaxies in a cluster can retrieve its mass quite accurately.

A schematic view of the architecture used in this work can be seen in Fig. 1

The training is done over a training set consisting of 10273 galaxy subsets with their respective inner slopes, which act as targets. The loss function to minimize during the training is the negative logarithmic likelihood of the training sample:

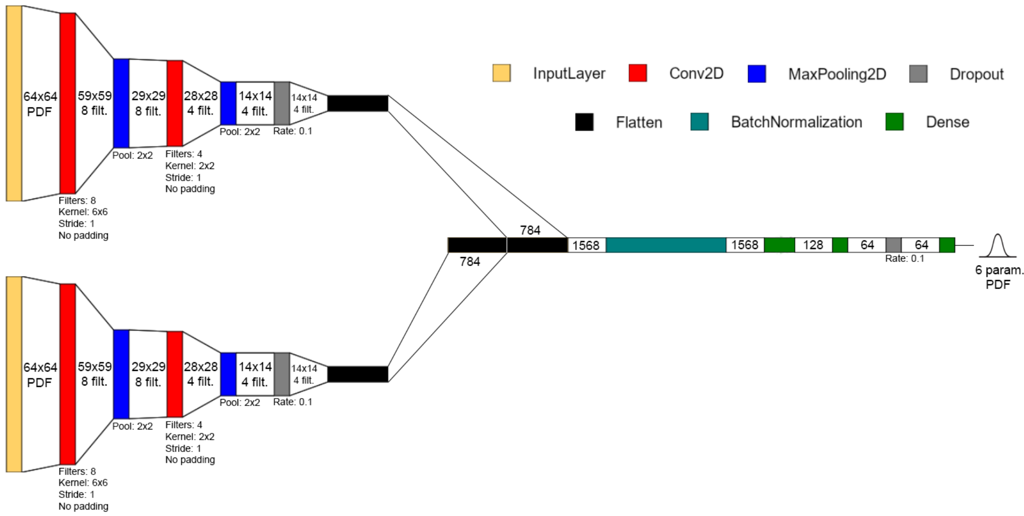

Figure 1: Schematic representation of our double channel MDCNN architecture to infer inner slopes of the DM profiles (slope at 150 pc) of galaxies from their 2D phase-space mappings of positional and dynamical distributions of stars. The MDCNN extracts the spatial features from the phase-space mappings and gradually compresses into high-order features until describing the input with only 6 parameters, which are used as parameters of a double Gaussian corresponding to approximation of the posterior distribution of the inner slopes values.

$$p_g(x|\theta) = \sum_{j=1}^{2} \phi_j \mathbf{N}(x, \mu_j, \sigma_j), \tag{1}$$

where $\mathbf{N}(x, \mu_j, \sigma_j)$ is the j Gaussian with mean μ_j and standard deviation σ_j, ϕ_j is the weight of the j Gaussian, so that $\sum_{j=1}^{n_g} \phi_j = 1$, and θ are the 2D maps representing the PDFs of the phase space described above.

We have performed multiple complete training runs using only 10 galaxies as validation and test datasets in each one, changing the galaxies that would come out of the training dataset in each of the training runs to evaluate the network in several projections of every galaxy. This allows us to analyse the consistency of the model training and its performance in a large number of galaxies without compromising the training dataset. The amount of data and the constructed arquitecture allows these successive trainings to be carried out in a small number of hours by using a personal laptop.

The output posterior distribution represents the random or aleatoric uncertainty in the slope prediction of the final model, but it does not represent the uncertainty due to the stochastic nature of the weight determination while training the neural network (epistemic uncertainty), which can lead to different models for the same training conditions when dealing with limited data. We use the Monte Carlo dropout method (MC-Dropout) (Gal & Ghahramani, 2015) to approximate the epistemic uncertainty which is based on the repeated evaluation of the same input, randomly setting to 0 the weights on some layers while doing each inference, to construct a final evaluation with statistical information about the epistemic uncertainty. The constructed final posterior for each galaxy projection is the normalized mean of 100 multivariate Gaussian posteriors inferred by the model with active dropout layers.

3 Results

The inner slope of simulated galaxies is predicted with a mean absolute deviation of $\mu_\epsilon = 0.056$ (where the deviation is defined as $\epsilon = \gamma_{\text{Real}} - \gamma_{\text{Pred}}$, and the predicted inner slope, γ_{Pred}, is obtained from the mode of the PDFs), with 82% (98%) of the galaxies having their inner slope correctly determined within $\pm$ 0.1 (0.3) of their true value.

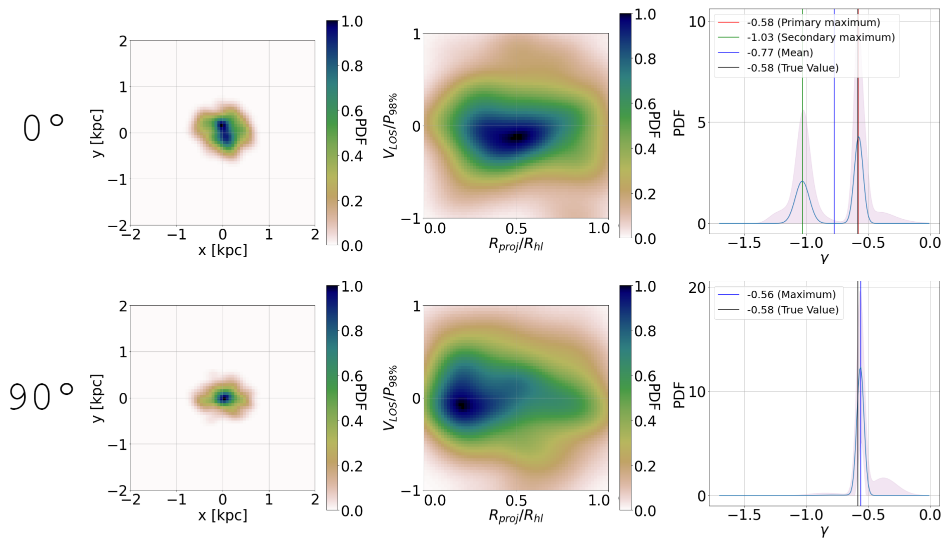

Figure 2: Probability density distributions used by the neural network as input in the case of one simulated galaxy subset seen at 0º (face-on) and 90º (side-on), alongside with the Bayesian posteriors predicted by the model. Left column: PDFs in the {x,y} phase space. Central column: PDFs in the $\{\hat{R}_{\rm proj}, \hat{v}_{\rm LOS}\}$ phase space. Right column: predicted Bayesian posterior in the space of inner slope of the DM profile (slope at 150 pc); shaded regions represent the standard deviation of the posterior values for the MC-Dropout inferences at each slope point. The red vertical line shows the primary maximum of the posterior distribution (the mode), the green one the secondary maximum and the black one the true value of the inner DM slope. As a blue line, the mean of the distribution is shown. This example shows how the appearance of double peaks in the posterior distributions is strongly related to the viewing angle.

The posteriors PDFs have a mean standard deviation of $\sigma_{\rm pos} = 0.108$, showing no bias towards more accuracy for cuspy or cored galaxies. Most of the posteriors for the different projections have an approximately normal distribution (the second Gaussian disappearing or constituting a skewness correction to the main Gaussian), but several of them have two distinct peaks. Specifically, around 30% of the galaxies have double peaks in more than 10% of their posteriors, but all of them present a roughly monomodal posterior in some projections. This indicates that the appearance of double peaks is a consequence of the fact that some information on the underlying DM profiles is hidden when viewing the galaxy at some particular angle, while it is released and efficiently passed to the network when looking at the galaxy from other angles (Fig. 2).

We adopt the catalogs by Walker et al. (2009) to directly compare our results with those obtained using the code GravSphere, as in Read et al. (2019). The selected galaxies are Carina, Sextans, Fornax and Sculptor, for which we further use the center position, velocity, ellipticity and half-light radius as compiled in Battaglia et al. (2022). We do not take observational uncertainties into account in the results presented in this work, since adding noise to the data by making use of uncertainties in the line-of-sight velocity only goes so far as to alter the mean, maximum and width of the posteriors by an order of 10^{-2} over multiple iterations for these four galaxies, making the uncertainties negligible. Our model recovers their inner slopes yielding values consistent with those obtained in Read et al. (2019) (Table 1). We found that the Fornax dSph has a strong indication of having a central DM core, Carina and Sextans have cusps (although the latter with a large uncertainty), while Sculptor shows a double peaked PDF indicating that a cusp is preferred, but a core can not be ruled out. These results are in agreement with several previously derived inner slopes for these galaxies (e.g. Walker & Peñarrubia, 2011; Brook & Di Cintio, 2015; Pascale et al., 2018; Richardson & Fairbairn, 2014) which confirms that our model trained on simulations can be reliably applied to observations.

	$\gamma_{\rm GS}$	$\gamma_{\rm NN}$
Carina	$-1.23^{+0.39}_{-0.35}$	$-1.06^{+0.05}_{-0.04}$
Sextans	$-0.95^{+0.25}_{-0.25}$	$-1.25^{+0.25}_{-0.09}$
Fornax	$-0.30^{+0.21}_{-0.28}$	$-0.38^{+0.01}_{-0.02}$
Sculptor	$-0.83^{+0.30}_{-0.25}$	$-1.08^{+0.08}_{-0.04}$

Table 1: Inner slope of the DM profile (at 150 pc) for Carina, Sextans, Sculptor and Fornax galaxies predicted by GRAVSPHERE ($\gamma_{\rm GS}$) and our neural network ($\gamma_{\rm NN}$), with their 68 per cent confidence intervals (for the neural network posterior, taking the primary maximum as reference).

4 Conclusions

We have shown that deep learning techniques provide a innovative method for the determination of the inner DM profiles in dwarf galaxies, complementary to the use of Jeans and Schwarzschild modelling, achieving great accuracy and offering a complex representation of uncertainties.

In the future, the architecture of this model could be expanded by including more input data,proper motion of stars from missions like *GAIA* (Gaia Collaboration et al., 2021). The current architecture could also be used as a basis for building models that provide a more complete output, such as a prediction of the full density profile of galaxies.

Our newly developed neural network method is a promising tool for the study of the mass distribution within dwarf galaxies, which in turn can help discriminate between different models and, in such, constraining the properties of the DM.

Impact statement

Our results show that simulation-based inference with neural networks provide a innovative and complementary method for the determination of the inner matter density profiles in galaxies. The new methodology shows much potential both in the accuracy of inferences and in the ability to properly construct its uncertainties, which is extremely important because the determination of the mass density profiles of galaxies is extremely closely linked to our ability to constrain the properties of the elusive dark matter.

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Checklist

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes]
   (b) Did you describe the limitations of your work? [Yes] The network only accepts positional and line-of-sight velocity data, when other potential sources of information, such as proper velocity, are currently available in some cases. It also infers only the internal slope instead of the full density profile. In addition, the training dataset still has a lot of potential to be extended and improved.
   (c) Did you discuss any potential negative societal impacts of your work? [No] It does not have any.
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes] It does.
2. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [No] The code is proprietary for now.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes]
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] A complete analysis of errors is done in the work. In this paper only give direct information about the methodology and the errors related to the probability distributions of the MDCNN.
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No] Irrelevant. The scope of this work and size of used data allows calculations to be performed easily on personal laptops.
3. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes]
   (b) Did you mention the license of the assets? [No] They are simulations given to us by their creators who are coauthors of this work.
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   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [Yes] They are simulations given to us by their creators who are coauthors of this work.
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