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# Domain Adaptation for Simulation-Based Dark Matter Searches with Strong Gravitational Lensing

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## Abstract

The application of machine learning for quantifying dark matter substructure is growing in popularity. However, due to the differences with the real instrumental data, machine learning models trained on simulations are expected to lose accuracy when applied to real data. Here, domain adaptation can serve as a crucial bridge between simulations and real data applications. In this work, we demonstrate the power of domain adaptation techniques applied to strong gravitational lensing data with dark matter substructure. We show with simulated data sets representative of Euclid and Hubble Space Telescope (HST) observations that domain adaptation can significantly mitigate the losses in the model performance when applied to new domains.

## 1 Introduction

One of the great achievements of astrophysics in the last century was the realization by Zwicky, Rubin and others that the observed baryonic mass (stars, galaxies, etc.) was not consistent with the dynamics of galaxies and clusters. A natural solution to this problem was to consider unseen dark matter compensating for this discrepancy. Presently, all efforts aimed at extracting a non-gravitational signature of dark matter have come up empty. While this does not mean that dark matter can not communicate with Standard Model (SM) particles, as its SM couplings may be strongly suppressed, there is also the possibility that such interactions do not exist. Since its discovery, subsequent evidence for particle dark matter from its coupling to gravity is almost irrefutable [1, 2, 3]. A particularly sensitive probe is strong gravitational lensing [4, 5, 6] for which we restrict ourselves in this article.

Strong gravitational lensing has already seen some promising success in extracting information about dark matter substructure. More recently, there has been a plethora of applications of machine

learning to this challenge, ranging from classification [7, 8, 9], regression [10], segmentation analysis [11, 12], and anomaly detection [13]. To date, all works have exclusively focused on the application of these techniques to simulations, in large part due to the limited availability of strong lensing data; something that is anticipated to change in the near future with the commissioning of the Vera C. Rubin Observatory and the launch of Euclid [14, 15]. However, naively applying models trained on simulations to real data will not likely to be successful, as the data idiosyncrasies will significantly diminish the accuracy of the model. A promising method to bridge the gap between a model trained on simulations and real data is based on the technique of domain adaptation (DA) [16]. A subset of transfer learning, domain adaptation is focused on the generalization of the model across different domains or data sets drawn from different underlying distributions. The goal of domain adaptation is to adapt a model trained on one data set (source) by generalizing it to another domain (target), where the objective of the model is unchanged. In practice, domain adaptation can be realized in several ways, including supervised, semi-supervised, and unsupervised approaches [17, 18, 19].

In this work, we consider domain adaptation for dark matter searches in strong gravitational lensing. With the present lack of sufficient real data, we use two data sets to realize mock observations with different surveys, HST and Euclid, of galaxy-galaxy strong gravitational lensing to carefully test the performance of domain adaptation prior to its applications to real data. We evaluate the models trained on the source data set to identify various types of dark matter substructure on the target data set. We compare the performance of two domain adaptation algorithms based on convolutional neural networks and equivariant neural networks that incorporate a known group symmetry to enhance performance.

## 2 Dark Matter Detection and Strong Gravitational Lensing

The  $\Lambda$  Cold Dark Matter ( $\Lambda$ CDM) model envisions near-scale invariant density fluctuations, present in the early universe, serving as seeds of large scale structure via hierarchical structure formation. Structures such as dark matter halos are formed from the coalescence of smaller halos [20]. Evidence for such merges has been observed in our Galaxy [21, 22, 23] and is a general prediction of N-body simulations where evidence of mergers should remain largely intact. Comparison between simulation and observation indicates good agreement with  $\Lambda$ CDM on large-scales [1, 2, 3]. However, discrepancies begin to arise on smaller, sub-galactic scales. These include the core-vs-cusp, too big to fail, missing satellite, and diversity problems.

Similar to [7, 13] we consider data sets of three substructure classes; no substructure, NFW subhalos of cold dark matter, and vortex substructure of superfluid type (axion) dark matter. We construct the simulations with `lenstronomy` [24] to mimic the characteristics of HST and Euclid using the default instrument and observational settings. For background sources we use images of galaxies from the Galaxy10 DECals data set [25] processed with a Gaussian mask and convolved with a Gaussian of size 2 pixels. This prevents the lensing of unwanted foreground sources and noise in the image. We choose the apparent magnitude of the background galaxy such that the the signal-to-noise ratio (SNR) of the lensing arcs are consistent with real lensing data –  $SNR \sim 20$  [26].

Domain adaptation requires at least two data sets, the source and the target. In this work we will demonstrate domain adaptation between models trained on mock HST observations and Euclid (and vice versa). Thus, it will be our goal to successfully adapt and evaluate the algorithms trained on a given source data set to the target data set.

## 3 Domain Adaptation

The goal of our work is to train a supervised model on a source data set and adapt it to a target data set. For this task we use a Convolutional Neural Network (CNN), specifically *EfficientNet* [27], as our base architecture. This is the same type of architecture that has achieved top performance in previous applications to lensing data sets [7, 13, 28]. More generally, CNNs are known to outperform other methods of classification for strong gravitational lenses [29], nonetheless, as noted by [30], a model trained on simulations can perform poorly on real data.

To improve the performance of models trained on simulated data, we use unsupervised domain adaptation, which attempts to mitigate the effects of the domain shift between the source and the target domains. It enables a transfer of knowledge gained from a labeled source data set to a distinct

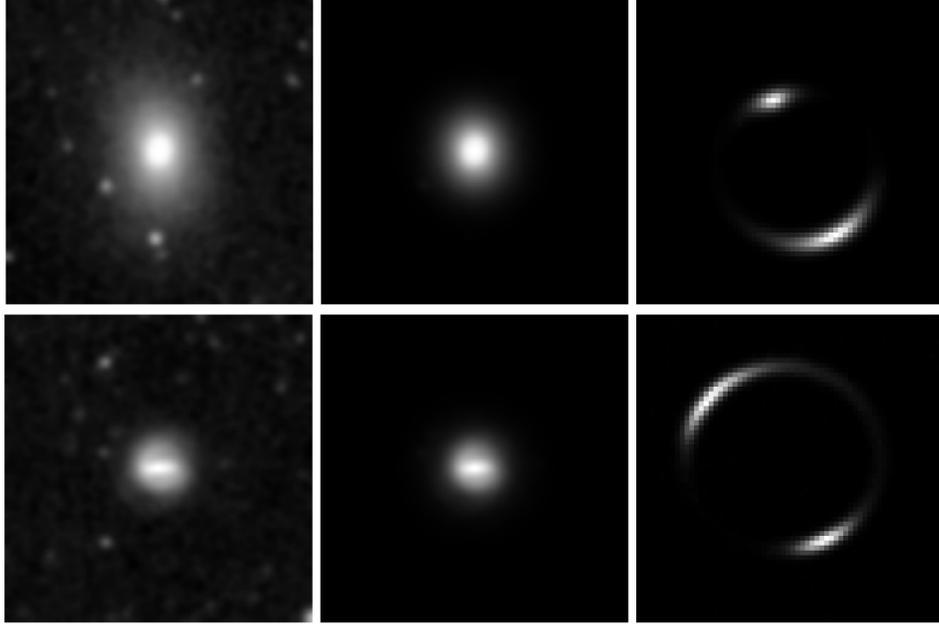


Figure 1: Example of lensing simulation of mock HST (top) and Euclid (bottom) data sets. Image of real galaxy from the Galaxy10 DECaLS data set (left), image processed with a Gaussian filter to remove unwanted background (center), and final lensed image (right).

unlabeled target data set, within the constraint that the objective remains the same [31]. Examples from each data set are shown in Figure. 1.

We utilize Adversarial Discriminative Domain Adaptation (ADDA) [32], an adversarial adaptation method with the goal of minimizing the domain discrepancy distance through an adversarial objective with respect to a discriminator. Ideally the discriminator will be unable to distinguish between the source and the target distributions. We consider that we have access to source images  $\mathbf{X}_s$  and labels  $\mathbf{Y}_s$  that come from a source distribution  $p_s(x, y)$  and also target images  $\mathbf{X}_t$  from a target distribution  $p_t(x, y)$ . Our objective is to learn a target encoder  $M_t$  and classifier  $C_t$  that classifies  $\mathbf{X}_t$  into  $K$  classes.

In addition to the baseline CNN models, we also consider an Equivariant Neural Network (ENN) [33] for substructure classification. ENNs can be thought of a generalization of a CNN that encode the representation of a useful symmetry, both global or local, such that its group convolutions are invariant symmetries present in the data. This is useful if there is a known symmetry. As we expect lensing images to have symmetries beyond simple translation, for example rotations, the flexibility of choosing different group representations is expected to improve the performance.

The ENN we use consists of a group equivariant convolutional neural network [34] with six equivariant convolution blocks. We utilize the dihedral group  $D_2$ , whose symmetry mappings include the identity, rotations by  $\pm\pi$  and horizontal/vertical reflections. Each block is composed of a convolutional layer, a batch normalization layer and a ReLU activation function. After each pair of layers we perform channel-wise average-pooling and in the end we use a fully connected layer for multiclass classification.

## 4 Results

We test the applicability of unsupervised domain adaptation using ADDA between mock HST and Euclid observations in the context of multi-class classification of three types of substructure: no substructure, NFW subhalos of CDM, and superfluid DM vortices. We employ two different base classifiers - *EfficientNet* and an ENN. For training we use 30,000 images for the source domain and 30,000 images for the target domain; in both cases there are 10,000 images per class. For

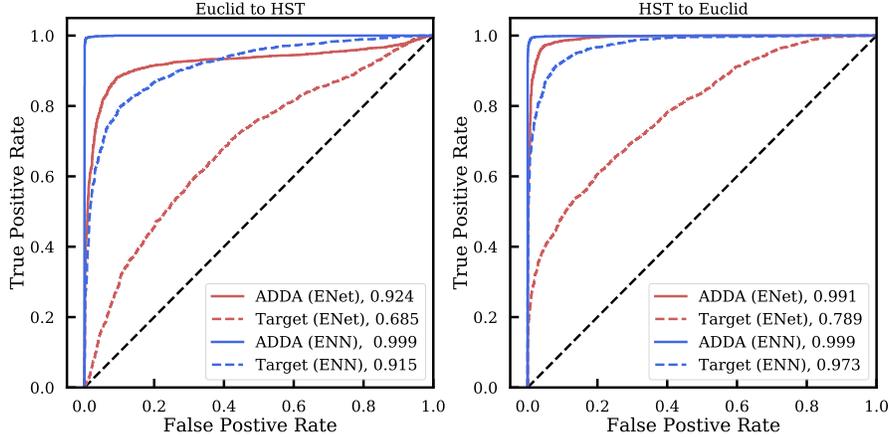


Figure 2: Application of ADDA for domain adaptation between two different mock simulations of strong lensing surveys: from Euclid to HST (left) and HST to Euclid (right). Dashed lines represent the naive application of classifier trained on source data set to target and solid represents result after training with ADDA. Red curves represent models based on *EfficientNet* and blue based on an ENN.

validation we use 7,500 images for the source domain and 7,500 images for the target; in both cases there are 2,500 images per class. We use the *Adam* optimizer [35] to minimize the loss. We trained both *EfficientNet* and the ENN for 200 epochs, training with a patience of 15 epochs, such that if the accuracy of the model does not improve in 15 epochs, the training is stopped. Learning rate, weight decay and other hyperparameters were optimized through a hyperparameter search. We have set the learning rate to  $2 \times 10^{-3}$  and weight decay to  $1 \times 10^{-5}$ . We utilize the area under the ROC curve (AUC) on the target validation set as the metric for classifier performance for all the models. All quoted AUC values are macro-averaged. All machine learning models were implemented using PyTorch [36] and are run on a single NVIDIA Tesla P100 GPU. The ROC curves for both combinations of source/target in addition to results for different architectures are presented in Figure 2.

We first train *EfficientNet* on the source data sets where it achieved a macro-averaged AUC  $\approx 0.999$  for both data sets and an accuracy of  $\approx 99.6\%$  for Euclid and  $\approx 99.5\%$  for HST. Applying these models to the corresponding target data sets naively, i.e. *without* domain adaptation, results in an AUC of  $\approx 0.685$  (0.789) when the Euclid (HST) model is applied to the HST (Euclid) data set and an accuracy of  $\approx 50\%$  (51%), a significantly degraded performance. This degradation is anticipated, even though the underlying physics is identical, due to different instrument systematics.

Following the application of unsupervised domain adaptation with ADDA, we observe a significant improvement in the application to the target data set. With the *EfficientNet* based algorithm we achieve a  $\approx 35\%$  improvement in AUC, at  $\approx 0.924$ , and accuracy of  $\approx 87\%$  adapting from Euclid to HST data sets. Going the other direction the AUC is improved to 0.991 and reaches an accuracy of 0.88%. This is a remarkable improvement in performance which can be further appreciated by comparing the ROC curves in Figure. 2. The red dashed curves correspond to the naive application of the *EfficientNet* and the solid curves correspond to the performance with ADDA. While there is still some room for improvement in DA from Euclid to HST (the left figure), DA from HST to Euclid results in near-perfect classification.

While *EfficientNet* saw a great improvement with ADDA, it is clear there is still a lot on the table as standard binary classification has an AUC near unity. This informs us that one should be able to further improve on the UDA performance, particularly in the case of going from Euclid to HST. A natural improvement to consider is changing the baseline classifier to an equivariant neural network. When we leverage extra, known symmetries of lensing we should see an appreciable bump in performance as our model does not need to learn this unimportant symmetry. In principal one could also consider the inclusion of other UDA but we will not consider that in this work.

Training and testing on source data sets the ENN achieves an AUC of  $\approx 0.999$  on both data sets and an accuracy of 99.4% and 99.7% for Euclid and HST respectively. The naive application to the

test data set (i.e. no domain adaption) again results in degraded performance, realized with an AUC for the Euclid (HST) trained model applied to HST (Euclid) data of  $\approx 0.915$  (0.973). This realizes a remarkable performance simply in the naive application of the ENN to the target data set. After training with ADDA we find that our models are then able to achieve effectively perfect classification with AUCs of  $\approx 0.999$  for both combinations of source/target and accuracy an of 99.1% (97.5%) from Euclid (HST) to HST (Euclid). We see that the performance with our ENN is near optimal, achieving a significant performance bump over the CNN. This is truly impressive as our DA algorithm is unsupervised – it never saw the labels from the target data set, yet was nearly perfect at adapting to the new domain. This is, of course, exactly the kind of transfer of knowledge one would hope to be able to do between simulations and real data sets.

## 5 Discussion & Conclusion

With the upcoming arrival of strong gravitational lensing data from Euclid and the Vera Rubin Observatory, it is imperative to assess how algorithms trained on simulations can be applied to real world data. In this work, we studied how unsupervised domain adaptation algorithms can be used to adapt a model trained on one set of data (the source) to another set of gravitational lensing data (the target). To make a quantitative comparison, we based our work on two sets of realistic lensing simulations, a mock data set from Euclid and another for HST. While we have restricted ourselves to substructure classification in this work, domain adaptation techniques can be additionally useful in the broader context of studying dark matter, from regression to image segmentation, in applications to real world strong gravitational lensing data sets.

### Broader Impact

This work serves to augment the understanding and application of machine learning in cosmology - which is still very much in its initial stages. This work serves to increase the accessibility to those interested in applications of machine learning for strong lensing applications around the globe as our simulation data set and analysis pipeline is open sourced. Given the computational requirements of our implementation, those who have limited access to computing power may be at a disadvantage.

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