Equivariant Neural Networks for Signatures of Dark Matter Morphology in Strong Lensing Data

Geo Jolly Cheeramvelil
Department of Computer Science and Engineering,
Amrita School of Computing, Amrita Vishwa Vidyapeetham, Amritapuri, India
amenu4cse20025@am.students.amrita.edu

Sergei Gleyzer
Department of Physics & Astronomy, University of Alabama,
Tuscaloosa, AL 35401, USA
sgleyzer@ua.edu

Michael W. Toomey
Center for Theoretical Physics, Massachusetts Institute of Technology,
Cambridge, MA 02139, USA
mtoomey@mit.edu

Abstract

One of the most promising avenues to study dark matter is from its interactions with gravity. In particular, it is well known that dark matter can be studied from the effect of its substructure in strong galaxy-galaxy lensing images. However, in practice, this is a very challenging problem to solve as the lensing signature is a sub-dominant effect, relative to that from the main halo, and there are also many systematics which are hard to account for. To circumvent these issues, machine learning has been studied extensively in the context of lensing to circumvent exactly these problems. Indeed, deep learning methods have the potential to accurately identify images containing substructure accurately. Most applications of machine learning to strong lensing rely on using convolution neural networks (CNN). In this work, we study the performance of equivariant neural networks (ENN) using simulated strong galaxy-galaxy lensing images as a means to study dark matter. We find that equivariant neural networks outperform state-of-the-art CNNs in both classification and regression tasks. This suggests that ENNs may be better suited for future lensing studies.

1 Introduction

Despite having now been studied for nearly a century, the identity of dark matter remains as elusive as ever. Constituting about 80% of the mass content in the Universe today, or roughly 25% of the current energy density, dark matter is one of the bedrocks of modern cosmology and astrophysics. While the evidence for dark matter is strong, e.g. it is required for the formation of the cosmic microwave background [1], experiments aimed at identifying its true nature have consistently seen no signal [2, 3, 4, 5, 6, 7]. While dark matter may well be one of the proposed models that have been extensively studied, e.g. weakly interacting massive particles (WIMPs), its could also be the case that dark matter may not couple to the Standard Model or that its couplings are too weak to be observed with direct or indirect detection.

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While it is certainly possible for dark matter to lack such couplings to the Standard Model, it should, in principle, be possible to shine light on the identity of dark matter via gravity alone. Indeed, since dark matter clearly does not violate the equivalence principle, it is worthwhile to consider avenues to understand and constrain dark matter from its gravitational interactions alone. A growing area of study in this direction has been the use of strong galaxy-galaxy lensing for the study of dark matter. Lensing is interesting in the context of dark matter because the extended lensing arcs are sensitive to the presence of substructure in the dark matter halo, see for example [8, 9, 10, 11, 12, 13, 14]. Changes to the distribution of dark matter (sub)halos and their morphology can then be leveraged to distinguish between different models. However, in practice the complexity of lensing data makes this a challenge.

To better harness the potential of strong gravitational lensing data, deep learning methods have been applied to both real and simulated data and emerged as powerful tools and capable of identifying substructures, e.g. [15, 16, 17, 18, 19]. Furthermore, some work has even suggested that, in the controlled setting of lensing simulations, ML methods have the power to distinguish between various dark matter models [20]. Most studies in this context have relied on the applications of state-of-the-art convolutional neural networks (CNN) like ResNet. Indeed, CNNs are well suited for applications to lensing data sets, as they are, in fact, images. The convolutional filters are able to leverage the local correlations in the structure of extended arcs while simultaneously taking advantage of the built-in transitional invariance of CNNs to reduce the complexity of training.

On another front, equivariant neural networks, such as E(2)-steerable Convolutional Neural Networks (CNNs), have gained prominence due to their ability to preserve further, inherent data symmetries, making them potentially well-suited for the diverse orientations and reflections present in strong gravitational lensing images. Equivariance in this context is rather simple, the architecture can be constructed in such a way that they are innately invariant to different discrete or continuous symmetries. This is expected to enhance performance as an architecture of a different design would be forced to learn such symmetries if they are present in the data. Concrete examples of this have been demonstrated in the literature. As an example, [21] investigated rotational equivariance when applied to spherical images. The authors demonstrate that non-equivariant CNN models necessitate substantial data augmentation to attain performance levels comparable to their smaller equivariant counterparts. Furthermore, the authors showcased the limitations of non-equivariant semantic segmentation models, revealing performance plateaus even with increased data augmentation. Notably, the result underscores that equivariant models not only outperform non-equivariant models but also achieve comparable performance with reduced training times – as one would naively expect.

In this work, we delve into the application of such equivariant neural networks for extracting information about dark matter from simulated galaxy-galaxy strong lensing images. In what follows we 1) explore the classification performance of our architectures for distinguishing between three different example dark matter models and 2) regress directly on the particle mass for ultra-light axion dark matter from the lensing images. We find that the equivariant neural networks are effective at increasing the performance on both tasks relative to traditional CNNs.

2 Methods

In this section, we describe the methodologies and approaches employed in our research to address the objectives outlined in the introduction. First we outline the details of the data set that we use in our analysis, which we follow by detailing the architectures that we have used. During training, we employ NVIDIA A100 GPUs to efficiently train all the deep learning models. These GPUs are crucial for handling the computational demands of training these complex models. We also use the Adam optimizer [22] and the cross-entropy loss function. Our training regimen spans 10 epochs, with each epoch processing batches of 64 samples. Additionally, we set the learning rate to 1×10^{-4}.

2.1 Data sets

For our analysis we construct two data sets. We call them Model A and Model B which are designed to mimic mock simulations of galaxy-galaxy strong lensing from observations with a Euclid or HST like survey, respectively. Our simulations which are produced with the package `lenstronomy` [23] create single channel images of size 64×64. We model the background galaxies, which are subsequently lensed, with with a Sersic light profile. For dark matter, we create three concrete classes.

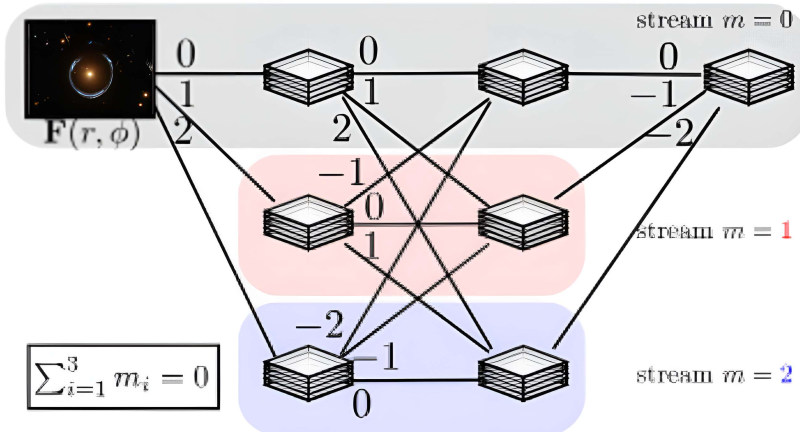

Figure 1: Example of a Harmonic Network with two hidden layers. Each horizontal stream represents a series of feature maps of constant rotation order, while the edges represents cross-correlations.

Table 1: Classification results from equivariant and baseline architectures for Models A and B.

Dataset	Model name	Accuracy	AUC
Model A	ResNet50	96.86	0.99740
	C8Steerable CNN	**99.02**	**0.99967**
	Harmonic Net	90.95	0.98516
	Equivariant transformer	92.413	0.99321
Model B	ResNet50	98.67	0.99926
	C8Steerable CNN	**99.28**	**0.99972**
	Harmonic Net	90.24	0.98904
	Equivariant transformer	97.42	0.99770

First we have lensing from standard cold dark matter (CDM) where our main halo, which we model with a spherical isothermal profile, has dark matter subhalos drawn from the the standard subhalo mass distribution (see [15] for more details). In addition to CDM, we also simulate the effects from a ultra-light axion dark matter model. In particular, our axion simulation corresponds to the regime where the particle mass $\sim 10^{-23}$ eV where substructure is high suppressed and the main observable is topological defects in the dark matter halo; namely vortex substructure [24]. Our last dark matter class consists of one with the absence of any substructure. While not a realistic model, in principle, since we are working with simulations it serves as a useful testbed for understanding the performance of our machine learning models – particularly because its signature is not expected to be degenerate with CDM or the axion. In compiling our simulations we construct 30,000 images per class and we separately construct a test set comprised of 5,000 images per class.

2.2 ResNet50

The first architecture that we study is the widely-used ResNet50 [25] architecture, which is known for its strong performance in various computer vision tasks. In particular, ResNet has been shown to exhibit very strong performance in previously strong lensing studies, e.g. [20]. In this work, we use ResNet50 as our reference point to compare to the performance of our equivariant network.

2.3 General E(2) - Equivariant Steerable CNNs

Next, we consider E(2) - Equivariant Steerable CNNs [26], a type of convolutional neural network architecture tailored to address the limitations of standard CNNs with added equivariance under the Euclidean group E(2), encompassing transformations like rotations and reflections in a two-dimensional Euclidean space. These networks employ mathematical techniques to decompose kernel constraints into irreducible subspaces, utilize group representations to steer features, employ a group

Table 2: Regression results from equivariant and baseline architectures for Models A and B.

Dataset	Model	RMSE	MSE	MAE
Model A	ResNet50	0.01309	1.31E-02	0.01030
	C8Steerable CNN	0.02504	6.30E-04	0.02096
	Equivariant Transformer	**0.00691**	**4.78E-05**	**0.00690**
	Harmonic Network	0.02181	7.88E-04	0.02181
Model B	ResNet50	0.01685	2.87E-04	0.01306
	C8Steerable CNN	0.00453	2.07E-05	**0.00357**
	Equivariant Transformer	**0.00410**	**1.69E-05**	0.00407
	Harmonic Network	0.01068	1.16E-04	0.00818

restriction operation, and include specific implementation details. This design ensures that feature maps exhibit consistent behavior under various transformations, making steerable CNNs a powerful tool for handling a broader range of geometric transformations in image processing tasks.

2.4 Equivariant Transformer

Equivariant Transformers[27] are a family of neural network layers designed for image-to-image mappings in computer vision tasks. They enhance model robustness by considering pre-defined continuous transformation groups like translations, rotations, and scaling. ETs incorporate prior knowledge about invariances to these transformations, enabling them to recognize and handle these transformations explicitly. They use specially-derived canonical coordinate systems and functions that are equivariant by construction, making them particularly useful for normalizing image appearance before subsequent operations like classification within convolutional neural networks, ultimately improving model performance in tasks involving transformational variations.

2.5 Harmonic Network

Harmonic Networks [28] or H-Nets are a type of convolutional neural network (CNN) that exhibits equivariance to patch-wise translation and 360-rotation, which is not the case for regular CNNs, where global rotation equivariance is typically sought through data augmentation. They achieve this by using circular harmonics instead of regular CNN filters, which return a maximal response and orientation for every receptive field patch. It works by creating different streams of constant rotation order responses which runs through the network - see Fig. 1. H-Nets use a rich, parameter-efficient, and low computational complexity representation, and deep feature maps within the network encode complicated rotational invariants.

3 Results & Discussion

Our main results after training our architectures for classification are presented in Table 1. We see that Harmonic Nets have the worst performance for Model A(B) with an AUC of 0.985 (0.989) and accuracy of 91.0% (90.2%) and followed by the equivariant transformer at 0.993 (0.998) and 92.4% (97.4%). Relative to ResNet50, the performance is actually *worse*, as the former architecture has an AUC and accuracy for Model A(B) of 0.997 (0.999) and 96.9% (98.7%), respectively. However, we find that the C8Steerable CNN achieves the leading performance with near perfect scores in classification with 0.999 (1.000) AUC and 99.0% (99.3%) accuracy for Model A (B). This appreciable gain in performance relative to ResNet50 can likely be attributed to the increased redundancies built into the architecture to learn representations of the data sets.

To complement the classification results, we also study the performance of these models in the context of regressing the axion mass. That is, we train the same architectures to predict the axion mass for the ultra-light dark matter sample based on the images. As Table 2 shows, the equivariant architectures consistently achieved lower RMSE, MSE, and MAE compared to non-equivariant models. Interestingly, it is the equivariant transformer that performs best at this regression task.

In this work, we have shown that equivariant neural networks, such as E(n) Steerable CNN and equivariant transformers, exhibited strong performance in both classification and regression tasks.

Notably, these equivariant networks also converged faster when compared to ResNet50. This faster convergence is a promising indicator of their efficiency and effectiveness in various machine learning applications for the identification and differentiation of dark matter substructure. We have demonstrated using simulations that the equivariant models perform well at distinguishing the lensing signature of CDM from other models such as ultra-light axion dark matter and a toy model with no dark matter substructure. We also saw remarkable performance relative to our CNN baseline in the context of regression. These results underscore the considerable potential of equivariant neural networks in enhancing our understanding of dark matter's underlying nature through strong gravitational lensing analysis.

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