Embedding Theoretical Baselines For Satellite Force Estimations

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Abstract

As satellites are progressively deployed to operate at lower orbital heights, the assumption of a constant drag for mission planning and satellite design will not hold up well. Expensive numerical simulation of satellite aerodynamic forces may be necessary to provide accurate estimations, especially at low altitudes (< 500km). To alleviate these data requirements in building surrogate models, a physics-informed pre-training strategy is explored to embed theoretical baselines within predictions. From the assumption of free-molecular flow, residual-learning of the rarefield aerodynamics can first serve as a form of low-fidelity approximation, before sparsely learning a corrector towards the ground truth. Under data-scarce conditions, the proposed approach outperformed models trained using only data or only physics, in terms of prediction accuracy.

1 Introduction

Over the years, the benefits and challenges surrounding satellite operation at very low Earth orbit (VLEO) are becoming better understood, and technological progress has gradually enabled its feasible implementation [3]. At such low altitudes, the influence of atmospheric density is no longer negligible, hence raising the need for accurate force estimations. While these can be realized through numerical simulations, the costs can be steep. In satellite mission planning and design, numerous sequential evaluations are required to accomplish orbit propagation over an operational lifetime. Conducting iterative shape and structural design over a single operating condition, or even optimized over the entire trajectory, compounds the computational demands. Various approaches have been explored to circumvent this, such as simplifying the geometry [2], optimizing in two-dimensions [4], selectively omitting physics [10], or developing general shaping strategies [5], with each presenting some form of trade-off in accuracy.

While surrogate modelling can relieve these costs, sufficient data is still required to construct accurate and generalizable prediction models. As such, a physics-informed pre-training strategy is explored to improve model accuracy under such data-scarce conditions. The use of machine-learning in enhancing physical analysis has been demonstrated to be achievable through careful integration [19, 21, 6]. In particular, pre-training strategies that tap on transfer-learning between related distributions have shown to be particularly effective in scientific tasks [9]. In this work, a theoretical baseline is provided to support the predictions of the ground truth satellite aerodynamic forces. By pre-training through residual-learning of a theoretical free-molecular flow, the aerodynamic forces can be approximated in a low-fidelity context and without any simulation data. The embedding of this physics-informed baseline enables a corrector to be learned from sparse simulation samples. The proposed approach is visualized in Figure 1.

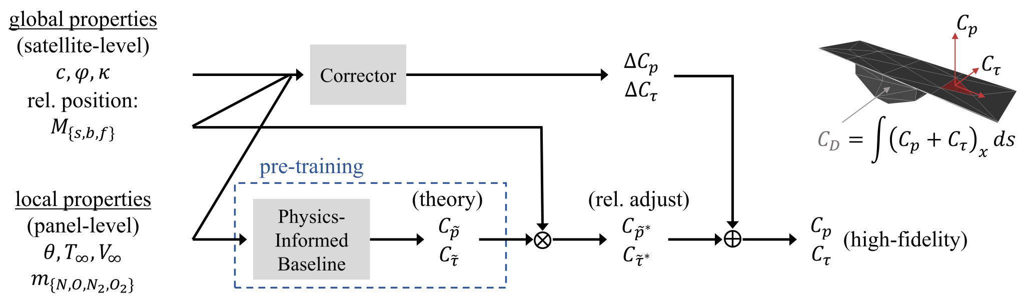

Figure 1: Schematic of the proposed embedding of a theoretical baseline to estimate surface forces of a satellite. By discretizing the satellite surface into a compilation of flat panels, a baseline model can be pre-trained to approximate local forces by directly minimizing the residuals describing free-molecular flat plate theory, without any ground truth samples. Masking the local panels using its relative position forms a globally-adjusted approximation. The corrector model is then trained from scarce samples to correct the approximation towards the ground truth distribution.

2 Methods

2.1 Numerical Experiments

Orbital Trajectory. As a toy problem, the satellite is projected on a circular, sun-synchronous orbit with a mean orbital altitude of 200 km, approximately 16 orbits per day, and inclination of 96.3° estimated using the J2 algorithm [8]. Only 24 hours of the flight path is used in the present analysis, and discretized into $N_t = 1440$ minutes of simulation data. The satellite moves at a constant orbital velocity V_∞ through each discretized point of the flight path, in which the environment is described by the atmospheric temperature T_∞, number density ρ_N, and mass fractions m_i of the four primary atmospheric constituents, i.e., $i = \{\mathrm{N}, \mathrm{O}, \mathrm{N}_2, \mathrm{O}_2\}$. These conditions are obtained from the NRLMSISE-00 atmospheric model [12] using referenced F10.7 solar flux and magnetic indices [20]. As visualized in Figure 2 , four arbitrary maneuvers are prescribed onto the flight path consisting of pitch φ and yaw κ rotations towards ±40° with each lasting around 35 minutes.

Satellite Forces Analysis. The ground truth satellite forces are determined using Direct Simulation Monte Carlo (DSMC) [1], which simulates the environment governed by the Boltzmann equation using representative gas particles with statistically sampled motion and collisions based on the kinetic theory of gases. A simplified form of the CYGNSS satellite [14] is used for this study, in which all surfaces are considered to be fully diffusive with a constant surface temperature of $T_w = 300\mathrm{K}$. Gas-surface interactions are characterized in DSMC using the Diffuse Reflection Incomplete Accommodation (DRIA) model [15] for an energy flux accommodation of $\alpha = 1$.

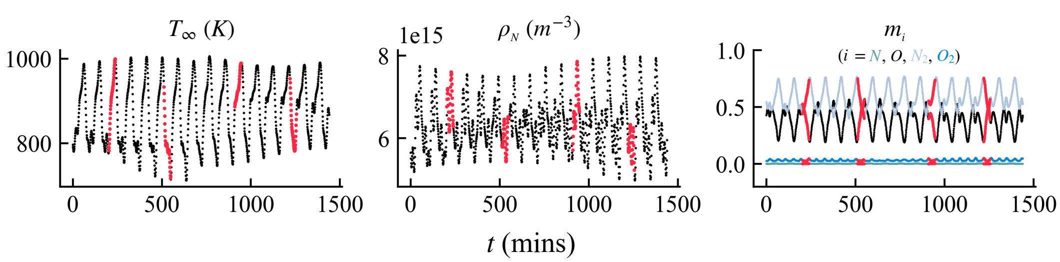

Figure 2: Orbital data corresponding to the full satellite path, with maneuvers highlighted in red.

2.2 Embedding Theoretical Baselines

Free-molecular flow theory. The forces acting on the satellite can be preliminarily approximated using the theory of free-molecular flow to form a physics-informed baseline. The atmosphere at high orbital altitudes can be highly rarefied with mean free path of the gas particles being significantly greater than the characteristic length of the satellite, thereby justifying a collisionless environment (negligible inter-particle collisions). The forces imparted on the satellite are effectively determined by the interaction of the undisturbed flow with the surface. From these simplifications, the rarefied aerodynamics of flat surfaces can be described theoretically [16], which includes an α-parameterization of its gas-surface interactions [11]. Thus, any satellite geometry can be discretized into a compilation of flat panels, each oriented at angles θ towards the flow with individual force contributions of

$$C_{\tilde{l}_i} = G_i Z_i \sin\theta + \frac{V_{(\mathrm{re}/\infty)} \sin\theta}{2} \left(\sqrt{\pi} Z_i \cos\theta + P_i\right) \tag{1}$$

$$C_{\tilde{d}_i} = \frac{P_i}{\sqrt{\pi}} + Q_i Z_i \cos\theta + \frac{V_{(\mathrm{re}/\infty)} \cos\theta}{2} \left(\sqrt{\pi} Z_i \cos\theta + P_i\right) \tag{2}$$

where $G_i = \frac{1}{2S_i^2}$, $P_i = \frac{1}{S_i} \exp\left(-S_i^2 \cos^2\theta\right)$, $Q_i = 1 + G_i$, and $Z_i = 1 + \operatorname{erf}\left(S_i \cos\theta\right)$, for a speed ratio between the bulk velocity to the most-probable thermal velocity as $S_i = \frac{V_\infty}{\sqrt{2kT_\infty/m_i}}$. The error function is defined as $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-y^2\right) dy$, and the velocity ratio follows the wall temperature-corrected derivation as $V_{(\mathrm{re}/\infty)} = \sqrt{\frac{1}{2}\left[1 + \alpha\left(\frac{4RT_w}{V_\infty^2} - 1\right)\right]}$ [7]. The Boltzmann constant is defined as k and the molar gas constant defined as R. The contributions to the surface-normal pressure and surface-tangential shear stress coefficients are reconstructed from the local panel's theoretical lift and drag properties as $C_{\tilde{p}_i} = C_{\tilde{d}_i} \cos\theta + C_{\tilde{l}_i} \sin\theta$ and $C_{\tilde{\tau}_i} = C_{\tilde{d}_i} \sin\theta - C_{\tilde{l}_i} \cos\theta$.

Physics-Informed Baseline (PI-B). A baseline model is pre-trained on free-molecular flow theory using a physics-informed approach [13], to provide an interpretable approximation of the ground truth. From a weighted-sum using the mass fractions of the four primary atmospheric constituents, the resulting approximation for the pressure and shear stress coefficients of the panel can be obtained. The **PI-B** learns this theoretical approximation through residual-learning of

$$f_{\tilde{p}} \coloneqq \hat{C}_{\tilde{p}} - \sum_i^4 m_i C_{\tilde{p}_i} \quad \text{and} \quad f_{\tilde{\tau}} \coloneqq \hat{C}_{\tilde{\tau}} - \sum_i^4 m_i C_{\tilde{\tau}_i} \tag{3}$$

where $\hat{C}_{\tilde{p}}$ and $\hat{C}_{\tilde{\tau}}$ are the output predictions by **PI-B** for each theoretical panel. The model undergoes pre-training by minimizing the loss function defined as $\mathcal{L} = \mathbb{E}_{x_{\text{PI-B}} \sim \mathcal{P}}\left[{f_{\tilde{p}}}^2 + {f_{\tilde{\tau}}}^2\right]$, in which inputs to the baseline model are $x_{\text{PI-B}} = \{\theta, T_\infty, V_\infty, m_{\mathrm{N}}, m_{\mathrm{O}}, m_{\mathrm{N}_2}, m_{\mathrm{O}_2}\}$ and sampled from within the respective bounds defined by the satellite flight path.

In the absence of simulation data, a theoretical free-molecular flow can provide a physically-grounded approximation for the aerodynamic forces. While useful, there are apparent shortcomings arising from these simplifications. For instance, the influence of surface concavity is absent as every particle is reflected only once; local Knudsen numbers can be small at VLEO, hence the frequency of inter-particle collisions can be considerable; the flow may not be hyperthermal depending on the environment; and the lack of flow-shielding modelling. By including a shadow analysis [17, 18], the latter can be better accounted for, albeit building on the hyperthermal flow assumption. Based on a panel's global orientation, its theoretical force contribution can be logically masked: if the panel is located in the flow-shadow of other panels, its force contribution is zero (M_s); if the panel is back-facing, its force contribution is zero (M_b); if the panel is fully front-facing, its shear stress is zero (M_f). By adjusting for the global orientation of the theoretical panel using logical masks, a low-fidelity baseline prediction of the ground truth can be obtained as

$$\hat{C}_{\tilde{p}^*} = M_s \cdot M_b \cdot \hat{C}_{\tilde{p}} \quad \text{and} \quad \hat{C}_{\tilde{\tau}^*} = M_s \cdot M_b \cdot M_f \cdot \hat{C}_{\tilde{\tau}} \tag{4}$$

where $\hat{C}_{\tilde{p}^*}$ and $\hat{C}_{\tilde{\tau}^*}$ are the output predictions of the globally-adjusted, physics-informed baseline, **PI-B(G)**.

Data-driven correction (Corr.) The ground truth force coefficients are expressed as a sum of the adjusted baseline output and a corrector, i.e., $C_p = C_{\tilde{p}^*} + \Delta C_p$ and $C_\tau = C_{\tilde{\tau}^*} + \Delta C_\tau$. By randomly sampling from the full satellite flight path, a sparse representation of the ground truth can be established. These sparse samples are used to learn the corrections ΔC_p and ΔC_τ as a non-linear mapping from the inputs $x = \{\theta, T_\infty, V_\infty, m_\mathrm{N}, m_\mathrm{O}, m_{\mathrm{N}_2}, m_{\mathrm{O}_2}, M_s, M_b, M_f, c, \varphi, \kappa\}$. Apart from the baseline inputs $x_{\text{PI-B}}$ and masks M_s, M_b, M_f, additional inputs include the panel baricenter c as a global identifier, and the prescribed pitch and yaw rotations. The full prediction model, **PI-B(G) + Corr.**, consists of the pre-trained baseline along with the corrector trained on sparse samples.

Training Procedure. In the current work, simple multi-layer perceptrons are used as the neural architecture for the full model, **PI-B(G) + Corr.**, and its sub-components. The model consists of four hidden layers with 64 neurons each, in which two belong to **PI-B** and another two to **Corr.** with ReLU activation functions interspersed. For the data-driven components, 3% and 10% of the full flight path is randomly sampled to form the training and test set, respectively. No ground truth simulation data is required for physics-informed pre-training. For comparison, the physics-informed pre-training involved a training cost equivalent to $\sim$2.5 converged simulation instances, which would correspond to 0.17% of the full flight path dataset. For the subsequent analysis in Section 3, the full flight path is considered. All network parameters are optimized up to a fixed number of steps with the ADAM optimizer and learning rate of 10^{-4}. The pre-trained weights of **PI-B** are kept fixed during the data-driven training. Fully data-driven models, **Direct**, are developed without pre-training using the same training sets, and contain similar network parameter sizes for comparison.

3 Results and Discussion

Performance Evaluation. In Figure 3, the local force coefficients of C_p and C_τ are integrated to obtain the satellite drag coefficients C_D across the entire 24-hour flight path. The model **PI-B(G)** tends to over-predict the drag while **Direct** under-predicts, with both exhibiting substantial deviations. Notably, the latter performs much worse than the former in modelling the maneuvers, under data-scarce conditions. The **PI-B(G) + Corr.** reconciles the two models to best reflect the ground truth. In Table 1, the mean absolute percentage error is computed as $\mathbb{E}\big[|(C_D - \hat{C}_D)\ /\ C_D| \times 100\%\big]$ over ten randomly-initialized states, of which the flight path on and off maneuvers (MVR.) are also considered separately. Based on the full flight path, the proposed approach achieved error reductions by approximate factors of $0.5\times$ and $0.4\times$ from the models trained using only data and only physics, respectively. However, the extension of **Corr.** reveals a trade-off in accuracy when on and off maneuvers.

Limitations. This preliminary investigation uses simple neural architectures to understand the potential and feasibility of embedding pre-trained physics models, to alleviate data requirements for surrogate modelling of satellite forces. As such, it will be meaningful to consider more complex architectures and frameworks to assess how well the results hold, especially with scale. By propagating the satellite with a variable drag over longer time frames, the need for accurate drag prediction will be better reflected since the position errors are expected to reduce substantially compared to conventional constant drag models.

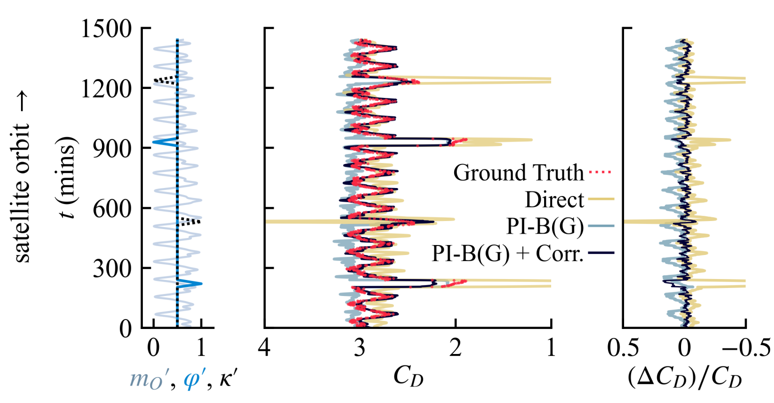

Figure 3: Evaluation of satellite C_D predictions over the full 24-hour flight path. In general, the proposed approach (**PI-B(G) + Corr.**) better reflects the ground truth than the pre-trained baseline (**PI-B(G)**) and the model without pre-training (**Direct**).

Table 1: Absolute percentage error in C_D predictions (%)

	Satellite Path		
Model	Full	On MVR.	Off MVR.
Direct	7.64 ± 2.40	35.66 ± 31.08	4.57 ± 3.25
PI-B(G)	5.75 ± 0.26	5.47 ± 0.07	5.78 ± 0.29
PI-B(G) + Corr.	2.93 ± 0.09	8.85 ± 3.29	2.29 ± 0.11

4 Conclusion

In this work, a physics-informed pre-training strategy is performed to improve the estimation of satellite surface forces. In the pre-training strategy, residuals formulated from a theoretical free-molecular flow are minimized to first develop a low-fidelity baseline, followed by fine-tuning with scarce samples of the ground truth. The proposed approach demonstrates improvement in accuracy over data-only and pre-trained only models by approximately $0.5\times$ and $0.4\times$, respectively, over a prescribed 24-hour satellite flight path. This motivates further investigations into the use of physically-inductive bias within surrogate models for satellite design and planning.

Acknowledgments and Disclosure of Funding

This work was conducted during Benjamin Y. J.'s graduate studies under the support of the NUS-DSO Graduate Programme. The authors appreciate the guidance provided by Prof. Chan Wai Lee from Nanyang Technological University.

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